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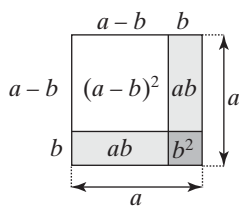
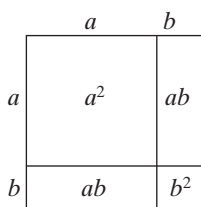
SPECIAL BINOMIAL PRODUCTS AND FACTORISATIONS

Algebraic Techniques



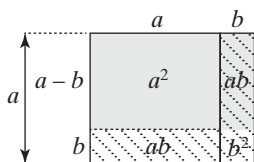
STUDY NOTES

- 1 In a **binomial product**, each term in the second binomial is multiplied by each term in the first binomial.
- 2 Finding the square of a binomial is just a **special binomial product**.



$(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$
 (The two ab rectangles must be subtracted from a^2 but this subtracts the square in the bottom right corner twice so b^2 must be added back on.)

- 3 To **square a binomial** 'square the first (term), square the last (term) and twice the product of the two'.
 For example, $(3x + 4)^2 = (3x)^2 + 2 \times 3x \times 4 + 4^2 = 9x^2 + 24x + 16$
 $(5k - 1)^2 = (5k)^2 - 2 \times 5k \times 1 + 1^2 = 25k^2 - 10k + 1$
- 4 Another special binomial product is the **sum by difference**. The product of a sum by difference will always be the difference of two squares.



[From a^2 we must subtract the bottom rectangle (ab) and add the side rectangle (ab) but without the bottom square (b^2) so $(a + b)(a - b) = a^2 - ab + ab - b^2 = a^2 - b^2$]

$$(a + b)(a - b) = a^2 - b^2$$

For example, $(x - 4)(x + 4) = x^2 - 4^2 = x^2 - 16$

$$(2x + 7y)(2x - 7y) = (2x)^2 - (7y)^2 = 4x^2 - 49y^2$$

- 5 Because **factorising is the opposite operation of expanding**, every special binomial product has an equivalent special factorisation.
- 6 The squares of binomials are called **perfect squares**.

$$a^2 + 2ab + b^2 = (a + b)^2 \quad a^2 - 2ab + b^2 = (a - b)^2$$

For example, $x^2 + 14x + 49 = (x + 7)^2$, $x^2 - 30x + 225 = (x - 15)^2$

- 7 If an algebraic expression consists of two terms both of which are squares and if one is subtracted from the other then we have the **difference of two squares**. When factorising the difference of two squares the result is the sum by difference:

$$a^2 - b^2 = (a + b)(a - b)$$

For example, $36 - x^2 = 6^2 - x^2$ so when factorised $36 - x^2 = (6 + x)(6 - x)$.

- 8 Some expressions involving four terms can be factorised by **grouping in pairs**. Find a common factor for the first 2 terms, and a different common factor for the last 2 terms. Then you may find that you have a new common factor for these two factorised expressions.

For example, $x^2 + ax + bx + ab = x(x + a) + b(x + a) = (x + a)(x + b)$

Checklist

Can you:

- 1 Find special products such as the square of a binomial and the sum by difference?
- 2 Recognise perfect squares?
- 3 Factorise quadratic expressions involving perfect squares and differences between two squares?
- 4 Expand and simplify more complicated algebraic expressions?



SPECIAL BINOMIAL PRODUCTS AND FACTORISATIONS

Algebraic Techniques



SKILLS CHECK

- 1 Expand and simplify:

a $(x+3)(x+8)$	b $(a-5)(a+9)$	c $(2a+3)(5b+4)$
d $(6x-1)(x-8)$	e $(5n+3)(2n-3)$	f $(x+y)(a+b)$
- 2 Fill in the blanks to complete these squares:

a $(x+3)^2 = x^2 + 6x + \underline{\hspace{2cm}}$	b $(x-6)^2 = x^2 - \underline{\hspace{2cm}} + 36$
c $(3x+2)^2 = 9x^2 + \underline{\hspace{2cm}} + 4$	d $(2x-5)^2 = \underline{\hspace{2cm}} - 20x + 25$
- 3 Find these squares of binomials:

a $(x+4)^2$	b $(x-9)^2$	c $(3a-10)^2$
d $(2p+3q)^2$	e $(7-x)^2$	f $(a+b)^2$
- 4 Fill in the blanks:

a $(p+3)(p-3) = p^2 - \underline{\hspace{2cm}}$	b $(2x-1)(2x+1) = \underline{\hspace{2cm}} - 1$
c $(3x+2y)(3x-2y) = 9x^2 - \underline{\hspace{2cm}}$	d $(x^2+2)(x^2-2) = \underline{\hspace{2cm}} - 4$
- 5 Find these products of a sum by difference:

a $(x-9)(x+9)$	b $(3x-1)(3x+1)$	c $(2n+7)(2n-7)$
d $(8-x)(8+x)$	e $(7x+2y)(7x-2y)$	f $(ab+2)(ab-2)$
- 6 Factorise:

a $x^2 + 13x + 42$	b $x^2 + 3x - 40$	c $x^2 - 10x + 16$	d $x^2 - 3x - 54$
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- 7 Complete:

a $x^2 + 8x + 16 = (x + \underline{\hspace{2cm}})^2$	b $a^2 - 10a + 25 = (a - \underline{\hspace{2cm}})^2$
c $n^2 + 2n + 1 = (n + \underline{\hspace{2cm}})^2$	d $x^2 - 16x + 64 = (x - \underline{\hspace{2cm}})^2$
- 8 Express as a perfect square:

a $x^2 + 20x + 100$	b $y^2 + 4y + 4$	c $49 + 14n + n^2$
d $a^2 - 2a + 1$	e $9 - 6m + m^2$	f $x^2 - 2xy + y^2$
- 9 Complete:

a $x^2 - 49 = (x+7)(x - \underline{\hspace{2cm}})$	b $a^2 - 9 = (a+3)(\underline{\hspace{2cm}})$
c $25 - m^2 = (\underline{\hspace{2cm}})(5-m)$	d $x^2 - y^2 = (x + \underline{\hspace{2cm}})(x - \underline{\hspace{2cm}})$
- 10 Factorise these differences of two squares:

a $x^2 - 81$	b $121 - y^2$	c $m^2 - 900$	d $p^2 - q^2$
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- 11 Find a common factor for the first pair of terms and a common factor for the second pair of terms and hence factorise the whole expression:

a $x^2 + xy + 2x + 2y$	b $ab + 3a - 5b - 15$	c $a^2 - ab - ac + bc$
d $2mn - 2m + 3n - 3$	e $xz - yz - x + y$	f $6kn - 3k - 2n + 1$



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Answers

1 a $(x+3)(x+8) = x^2 + 11x + 24$ b $(a-5)(a+9) = a^2 + 4a - 45$ c $(2a+3)(5b+4) = 10ab + 8a + 15b + 12$
 d $(6x-1)(x-8) = 6x^2 - 49x + 8$ e $(5n+3)(2n-3) = 10n^2 - 9n - 9$ f $(x+y)(a+b) = ax + ay + bx + by$
 2 a $(x+3)^2 = x^2 + 6x + 9$ b $(x-6)^2 = x^2 - 12x + 36$
 c $(3x+2)^2 = 9x^2 + 12x + 4$ d $(2x-5)^2 = 4x^2 - 20x + 25$
 3 a $(x+4)^2 = x^2 + 8x + 16$ b $(x-9)^2 = x^2 - 18x + 81$ c $(3a-10)^2 = 9a^2 - 60a + 100$
 d $(2p+3q)^2 = 4p^2 + 12pq + 9q^2$ e $(7-x)^2 = 49 - 14x + x^2$ f $(a+b)^2 = a^2 + 2ab + b^2$
 4 a $(p+3)(p-3) = p^2 - 9$ b $(2x-1)(2x+1) = 4x^2 - 1$
 c $(3x+2y)(3x-2y) = 9x^2 - 4y^2$ d $(x^2+2)(x^2-2) = x^4 - 4$
 5 a $(x-9)(x+9) = x^2 - 81$ b $(3x-1)(3x+1) = 9x^2 - 1$ c $(2n+7)(2n-7) = 4n^2 - 49$
 d $(8-x)(8+x) = 64 - x^2$ e $(7x+2y)(7x-2y) = 49x^2 - 4y^2$ f $(ab+2)(ab-2) = a^2b^2 - 4$
 6 a $x^2 + 13x + 42 = (x+6)(x+7)$ b $x^2 + 3x - 40 = (x+8)(x-5)$ c $x^2 - 10x + 16 = (x-2)(x-8)$ d $x^2 - 3x - 54 = (x+6)(x-9)$
 7 a $x^2 + 8x + 16 = (x+4)^2$ b $a^2 - 10a + 25 = (a-5)^2$
 c $n^2 + 2n + 1 = (n+1)^2$ d $x^2 - 16x + 64 = (x-8)^2$
 8 a $x^2 + 20x + 100 = (x+10)^2$ b $y^2 + 4y + 4 = (y+2)^2$ c $49 + 14n + n^2 = (n+7)^2$
 d $a^2 - 2a + 1 = (a-1)^2$ e $9 - 6m + m^2 = (3-m)^2$ f $x^2 - 2xy + y^2 = (x-y)^2$
 9 a $x^2 - 49 = (x+7)(x-7)$ b $a^2 - 9 = (a+3)(a-3)$
 c $25 - m^2 = (5+m)(5-m)$ d $x^2 - y^2 = (x+y)(x-y)$
 10 a $x^2 - 81 = (x+9)(x-9)$ b $121 - y^2 = (11+y)(11-y)$ c $m^2 - 900 = (m+30)(m-30)$ d $p^2 - q^2 = (p+q)(p-q)$
 11 a $x^2 + xy + 2x + 2y = x(x+y) + 2(x+y) = (x+y)(x+2)$ b $ab + 3a - 5b - 15 = a(b+3) - 5(b+3) = (b+3)(a-5)$ c $a^2 - ab - ac + bc = a(a-b) - c(a-b) = (a-b)(a-c)$
 d $2mn - 2m + 3n - 3 = 2m(n-1) + 3(n-1) = (n-1)(2m+3)$ e $xz - yz - x + y = z(x-y) - (x-y) = (x-y)(z-1)$ f $6kn - 3k - 2n + 1 = 3k(2n-1) - (2n-1) = (2n-1)(3k-1)$

SPECIAL BINOMIAL PRODUCTS AND FACTORISATIONS

Algebraic Techniques



INTERMEDIATE TEST

Part A Multiple Choice

- 1 $(2x - 5)^2 = ?$
A $4x^2 - 10x + 25$ **B** $4x^2 + 10x - 25$ **C** $4x^2 - 20x + 25$ **D** $4x^2 + 20x - 25$ (1 mark)
- 2 $(3x - y)(3x + y) = ?$
A $3x^2 - y^2$ **B** $3x^2 + y^2$ **C** $9x^2 - y^2$ **D** $9x^2 + y^2$ (1 mark)
- 3 $xy - 4x - 6y + 24 = ?$
A $(x - 6)(y - 4)$ **B** $(x - 6)(y + 4)$ **C** $(x + 6)(y - 4)$ **D** $(x + 6)(y + 4)$ (1 mark)
- 4 Which is a perfect square?
A $x^2 - 12x - 36$ **B** $x^2 + 34x + 289$ **C** $x^2 - 26x + 159$ **D** $x^2 + 28x - 196$ (1 mark)
- 5 $6k(k + 2) - (k + 2) = ?$
A $5k(k + 2)$ **B** $(k + 2)(5k - 1)$ **C** $6k(k + 2)$ **D** $(6k - 1)(k + 2)$ (1 mark)
- 6 When simplified $\frac{x^2 - 64}{8x - 64} = ?$ *Hint 1*
A $\frac{x}{8}$ **B** $\frac{(x + 1)(x - 1)}{8x - 1}$ **C** $\frac{x + 8}{8}$ **D** x (1 mark)
- 7 Which factorisation is correct?
A $x^2 - 49 = (x - 7)^2$ **B** $x^2 - 10x - 24 = (x - 4)(x - 6)$
C $x^2 - 24x + 144 = (x + 12)^2$ **D** $x^2 - 625 = (x + 25)(x - 25)$ (1 mark)
- 8 Which is not a perfect square?
A $x^2 + 40x + 400$ **B** $x^2 - 38x + 361$ **C** $x^2 + 42x + 441$ **D** $x^2 - 44x + 464$ (1 mark)
- 9 $(3x + 5)^2 - (3x - 5)^2 = ?$ *Hint 2*
A $18x^2 + 60x + 50$ **B** $18x^2 - 60x + 50$ **C** $18x^2 - 50$ **D** $60x$ (1 mark)

Part B Short Answer

- 10 Expand and simplify $(3x + 8)^2 + (x - 8)(x + 8)$ *Hint 3* (2 marks)
- 11 Factorise $a^2 + ab - a - b$ *Hint 4* (2 marks)
- 12 Find $a^2b[(2a + b)^2 - (2a - b)^2]$ *Hint 5* (2 marks)
- 13 Expand $(2a - 1)^2$ (1 mark)
- 14 **a** Factorise fully: $7p^2 - 63$ (2 marks)
b Factorise fully: $6x^2 - 48x + 6$ (2 marks)
- 15 **a** Find the value of $2x(x + 3) - (x + 2)(x - 2) - (x + 3)^2$ when:
i $x = 2$ **ii** $x = 4$ **iii** $x = -1$ (3 marks)
b By expanding and simplifying $2x(x + 3) - (x + 2)(x - 2) - (x + 3)^2$ show that it has the same value for every value of x . (2 marks)

Hint 1: Factorise both the numerator and denominator first.

Hint 2: Use the difference of two squares.

Hint 3: Find each binomial product separately and then add them together.

Hint 4: Be careful with the negative values. Remember that a divides into a once.

Hint 5: Be careful to subtract all of the second binomial product.

Your Feedback

$$\frac{\boxed{}}{25} \times 100\% = \boxed{}\%$$



SPECIAL BINOMIAL PRODUCTS AND FACTORISATIONS

Algebraic Techniques



ADVANCED TEST

1 Expand:

a $(4p + 5q)^2$

c $(5x - 3y)(5x + 3y)$

e $3x(x + 3)(x - 3)$

b $(12 - p)(12 + p)$

d $2(x - 4)^2$

f $(4x^2y^3 - 7z)(4x^2y^3 + 7z)$

(1 mark each)

2 Factorise:

a $81 - 4d^2$

c $4n^2 - 12n + 9$

e $x^4 + 4x^2 + 4$

b $36 - 12c + c^2$

d $x^4 - 25y^2$

f $p^2 - p + \frac{1}{4}$

(1 mark each)

3 Factorise fully:

a $2x^2 - 18$

c $x^4 - 1$

e $2x^3 - 12x^2 + 18x$

b $3x^2 + 30x + 75$

d $p - p^3$

f $16a^4 - 72a^2 + 81$

(1 mark each)

4 Expand and simplify:

a $(5x - 2)(7 - x) + (3x - 2)^2$

b $(x + y)(x - y) - (x - y)^2$

c $9(m + 5)(m - 2) - (3m - 10)(3m + 10)$

d $(5a - 2b)^2 - (5a + 2b)^2$

(2 marks each)

5 Factorise and simplify (leaving answers in factorised form if necessary):

a $\frac{9p^2 + 9}{9p^2 + 18p + 9}$

b $\frac{n^2 - 100}{n^2 - 12n + 20}$

c $\frac{x^3 - 4x}{x^3 - 4x^2 + 4x}$

d $\frac{x^2 - 2x + 1}{1 - x^2}$

(2 marks each)

6 Simplify:

a $\frac{1}{x^2 + 20x + 100} - \frac{1}{x^2 - 100}$

b $\frac{1}{a^3 - 1} \div \frac{1}{a^6 - 1}$

(2 marks each)

7 a Given that $1637^2 = 2\,679\,769$, explain why it is not difficult to find the answer to the product 1636×1638 without a calculator.

(1 mark)

b Find 897×903 without a calculator.

(1 mark)

Your Feedback

$\frac{\boxed{}}{40} \times 100\% = \boxed{}\%$



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SUBSTITUTION, EQUATIONS AND FORMULAE

Algebraic Techniques



STUDY NOTES

- 1 In an algebraic expression **pronumerals** are used as variables. A pronumeral is a symbol, often a letter, used in place of a number.
- 2 When **substituting** into an algebraic expression we replace the pronumerals with the numbers they represent. For example, if $m = 3$, $m + 6 = 3 + 6 = 9$, $2x$ means $2 \times x$, so when $x = 4$, $2x = 2 \times 4 = 8$
- 3 A **linear equation** is an equation involving a single power of an unknown (x , say). $3x + 7 = 19$, $8(2x - 5) - 4(x + 8) = 5(2x - 1)$ and $\frac{x+3}{4} = \frac{x-1}{5}$ are all examples of linear equations. Equations involving higher powers of an unknown (e.g. x^2 , x^7) are not linear.
- 4 Linear equations are **solved** by performing the same operation on **both sides** of the equation. An equation may be solved by:
 - **adding** the same number **to both sides** of the equation,
 - **subtracting** the same number **from both sides** of the equation,
 - **multiplying both sides** of the equation by the same amount, or
 - **dividing each side** of the equation by the same amount.
- 5 The **solution to an equation** is an algebraic expression, not a number. For example, the solution to the equation $x + 2 = 5$ is not 3, it is $x = 3$.
- 6 Always **simplify the expressions** on either side of the equals sign before attempting to solve.
- 7 Any equation involving fractions can be simplified by multiplying both sides of the equation by the **lowest common denominator**. Care must be taken to multiply every part of the equation by this common denominator. For example, if asked to solve $\frac{x}{4} + 5 = 2x$, every part must be multiplied by 4 (giving $x + 20 = 8x$).
- 8 The solution to an equation can always be **checked by substitution** into the original equation. If a value substituted into an equation makes the equation correct, we say the value **satisfies** the equation.
- 9 A **formula** is simply an algebraic expression connecting different values.
- 10 The **subject of a formula** is the value on the left hand side, before the equals sign. For example, the subject of the formula $A = 2\pi r^2 + 2\pi rh$ is A .
- 11 When substituting into a formula always **write the formula down first**, even if it is given in the question. Then **show the substitution** and finally **work out the answer**.
- 12 **Equations arising from substitution** into a formula are solved like any other equation after substituting the given values.
- 13 Many **problems** involve finding the answer to an equation. Think carefully about what is required and form an equation to solve.

Checklist

Can you:

- 1 Substitute values in algebraic expressions?
- 2 Solve simple linear equations?
- 3 Know how to check the solution to an equation?
- 4 Understand what is meant by a formula?
- 5 Substitute into a formula and solve equations arising from substitution into a formula?

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SUBSTITUTION, EQUATIONS AND FORMULAE

Algebraic Techniques



SKILLS CHECK

1 If $x = 4$, find the value of:

a $x + 3$

b $x - 6$

c $9 - x$

d $5x$

e $2x + 1$

f $3x - 2$

g x^2

h $2x^2$

2 If $a = 2$, $b = 5$ and $c = 6$, find the value of:

a $a + b$

b $c - b$

c $b + c - a$

d $3a$

e bc

f $ab + c$

g $3ac$

h $a^2 + b^2$

3 Solve:

a $x + 13 = 25$

b $y - 9 = 37$

c $5a = 325$

d $\frac{h}{6} = 12$

e $2x + 1 = 17$

f $3x - 4 = 5$

g $\frac{k}{2} + 3 = 7$

h $\frac{d-4}{3} = -9$

4 The given value of x satisfies the equation. True or false?

a $5x + 8 = 29 - 2x$ [$x = 3$]

b $\frac{x}{3} + \frac{x}{4} = 7$ [$x = 12$]

c $4(2x + 1) - 3(x + 5) = 28$ [$x = 8$]

d $\frac{7x + 1}{5} = \frac{2x - 9}{3}$ [$x = -4$]

5 Solve:

a $9p = 15 - p$

b $7k + 5 = 2k + 25$

6 If $V = Ah$, find V when $A = 120$ and $h = 12.5$

7 If $P = 2l + 2b$, find P when $l = 8$ and $b = 5$.

8 If $v = u + at$, find v when $u = 28$, $a = -4$ and $t = 5$.

9 If $c^2 = a^2 + b^2$ find c ($c > 0$), when $a = 24$ and $b = 45$.

10 If $A = \pi r^2$ find A , to one decimal place, when $r = 12$.

11 Given $m = \frac{y_2 - y_1}{x_2 - x_1}$, find m when $x_1 = -2$, $x_2 = 1$, $y_1 = 8$ and $y_2 = -4$.



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122–123

10 452.4 11 -4

Answers 1 a 7 b -2 c 5 d 20 e 9 f 10 g 16 h 32 i 2 a 7 b 1 c 9 d 6 e 30 f 16 g 36 h 29 i 3 a x = 12 b y = 46 c a = 65
d h = 72 e x = 8 f x = 3 g k = 8 h d = -23 i a true b true c false d false e a p = 1.5 b k = 4 c 6 d 1500 e 726 f 8 g 51
15 6

SUBSTITUTION, EQUATIONS AND FORMULAE

Algebraic Techniques



INTERMEDIATE TEST

Part A Multiple Choice

- 1 If $a = 5$ and $b = -3$ then $3a - b = ?$
A 12 **B** 18 **C** 32 **D** 38 (1 mark)
- 2 The solution to the equation $2(3x + 4) = 2x - 4$ is:
A $x = -2$ **B** $x = -3$ **C** $x = 2$ **D** $x = 3$ (1 mark)
- 3 If $x = -2$, then $3x^2 = ?$
A -36 **B** -12 **C** 12 **D** 36 (1 mark)
- 4 The formula $A = (n - 2) \times 180^\circ$ gives the sum of the angles of a polygon when n is the number of sides. What will be the sum of the angles of a 15-sided polygon?
A 345° **B** 1140° **C** 1980° **D** 2340° (1 mark)
- 5 Given that $y = mx + b$, and that $m = 2$ and $b = -6$, for what value of x will $y = 8$?
A $x = 0$ **B** $x = 1$ **C** $x = 4$ **D** $x = 7$ (1 mark)
- 6 Which is a correct rearrangement of the formula $v = u + at$?
A $u = at - v$ **B** $a = \frac{v - u}{t}$ **C** $t = \frac{u - v}{a}$ **D** $t = \frac{a}{v - u}$ (1 mark)

Part B Short Answer

- 7 Solve $6a + 9 - a = 8 - 3a + 1$ *Hint 1* (3 marks)
- 8 Solve $4(x - 5) + 3(x - 2) = 16$ *Hint 2* (3 marks)
- 9 Solve $\frac{2t + 4}{7} - 3 = 5$ (3 marks)
- 10 Solve $\frac{2m - 5}{3} = \frac{3m + 2}{5}$ *Hint 3* (3 marks)
- 11 **a** The given solution to the equation has at least one mistake. Between which lines do mistakes occur? Briefly justify your answer. *Hint 4* (4 marks)

$3x + 5 - 5x = 9$ *Line 1*
 $8x + 5 = 9$ *Line 2*
 $8x = 4$ *Line 3*
 $x = 2$ *Line 4*
- b** What is the correct solution to the equation $3x + 5 - 5x = 9$? (1 mark)
- 12 Young's Rule is a formula for changing an adult dose of medicine to a child's dose. Young's Rule is $D = \frac{yA}{y + 12}$ where D is the child's dose, y is the age of the child in years and A is the adult dose. Find the dose of medicine required for an 8-year-old child when the adult dose is 20 mL. (2 marks)

Hint 1: Collect like terms before attempting to solve.

Hint 2: Be careful to get the operation signs correct when expanding.

Hint 3: Multiply by the common denominator to get rid of the fractions.

Hint 4: There may be more than one mistake. Work carefully through each line of the solution.

Your Feedback

$\frac{\quad}{25} \times 100\% = \quad\%$



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123-124

PAGE 176

SUBSTITUTION, EQUATIONS AND FORMULAE

Algebraic Techniques



ADVANCED TEST

1 Solve:

a $5(x-3) + 2(x-7) = 6$

c $3(4p-5) = 2p - (7-p)$

e $\frac{3a}{4} + 7 = 2a + 1$

g $\frac{5x-3}{4} = \frac{4x-2}{3}$

i $\frac{3u+2}{5} - \frac{4u-1}{2} = \frac{2}{3}$

b $4(3k-2) = 3(1-2k)$

d $11(7-2g) - 5(3g-8) = 8g$

f $\frac{9n-3}{7} = 4n+5$

h $\frac{m}{5} + \frac{m}{3} = 16$

(2 marks each)

2 Solve:

a $x^2 = 0.25$

c $t^2 - 4 = 0$

b $9a^2 = 1$

d $\frac{y^2}{2} - 5 = 3$

(1 mark each)

3 a Solve $5x^2 - 2 = 7$ ($x > 0$), giving the answer correct to three decimal places. (2 marks)

b What difference does it make having the restriction that x is greater than zero? (1 mark)

4 If $p = -2$ and $q = -3$, evaluate:

a $5p - 3pq$

c $\frac{p+q}{p-q}$

b $4p^2$

d $pq^2 - p + q$

(1 mark each)

5 Given that $s = ut + \frac{1}{2}at^2$ find:

a s when $u = 32$, $a = 9.8$ and $t = 7$ (1 mark)

b a when $s = 1599$, $u = 58$ and $t = 13$ (2 marks)

6 Given that $V = \frac{4}{3}\pi r^3$ find, to two decimal places:

a V when $r = 9$ (1 mark)

b r when $V = 222\,665$ (2 marks)

7 The formula $F = \frac{9C}{5} + 32$ can be used to change temperatures in degrees Celsius to Fahrenheit.

a At what temperature should an oven be set in $^{\circ}\text{F}$ if a recipe states 180°C ? (1 mark)

b Tina read about a place where the temperature was 68°F . She thought that was very hot. Was she correct? Justify your answer. (1 mark)

8 a Given that $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ and that $d = 10$ when $x_1 = -3$, $y_1 = 2$ and $x_2 = 5$, find two possible values for y_2 . (2 marks)

b Given that the above formula finds the distance between the points (x_1, y_1) and (x_2, y_2) explain the significance of the above result. (1 mark)

Your Feedback

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FURTHER EQUATIONS AND INEQUALITIES

Algebraic Techniques



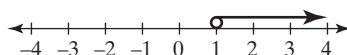
STUDY NOTES

- Many **problems** involve **finding the answer to an equation**. Think carefully about what is required and form an equation to solve. Let x (or some other variable) represent the unknown quantity and express the problem algebraically. Make sure that you answer the question that has been asked.
- When **solving equations to answer problems** we should always consider whether any answers are **physically possible**. For example, if finding the hypotenuse of a right-angled triangle, the length of a side of a triangle cannot be negative so we only choose the positive answer. Writing 'but $a > 0$ ' illustrates that you understand and have considered the negative value.
- An **inequality** is an expression that involves an inequality sign (such as $>$, $\leq$, $\neq$ for example) instead of an equals sign.
- There are different **inequality signs**, the meaning of which should be understood. A line (/) through the symbol means 'not', e.g. $\neq$ means 'is not equal to' and $\nlessgtr$ means 'is not greater than'.

Sign	Meaning
$>$	is greater than
$<$	is less than

Sign	Meaning
$\geq$	is greater than or equal to
$\leq$	is less than or equal to

- Inequalities** are solved in a similar way to equations. Perform the same operation on both sides of the inequality.
- An **inequality remains unchanged** if:
 - the same number is added to both sides of the inequality
 - the same number is subtracted from both sides of the inequality
 - both sides of the inequality are multiplied by a positive number
 - both sides of the inequality are divided by a positive number.
- If both sides of an inequality are multiplied (or divided) by a negative number the inequality does not remain unchanged. (For example, $3 > 2$ but if both numbers are multiplied by -1 , we get -3 and -2 and $-3 < -2$). **The inequality sign must be reversed when multiplying or dividing by a negative number.**
- The **solution to an inequality** is an algebraic expression. For example, $x > 3$ or $y \leq 9$. The solution is usually a range of values all of which will satisfy the inequality.
- Check by substituting** a value from the solution range into the inequality to see if it works. For example, if the solution to an inequality is $x > 3$, substitute $x = 4$ (or $x = 5$ or $x = 10$ or any value greater than three), to see whether it satisfies the inequality.
- The **solution to an inequality** can be graphed on a **number line**. The boundary point is shown with a circle. The circle is empty if the boundary point is not actually part of the solution (if the inequality sign is $>$ or $<$). The circle is solid if the boundary point is part of the solution (if the inequality sign is $\geq$ or $\leq$). A line is then drawn to show the direction of the set of values that satisfy the inequality. For example, to graph $x > 1$, place an empty circle at 1, and draw a line showing those values larger than one.



Checklist

Can you:

- Solve problems by forming equations and solving?
- Understand the need to consider whether answers are physically possible?
- Understand the different inequality signs?
- Solve linear inequalities?
- Graph inequalities on a number line?


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FURTHER EQUATIONS AND INEQUALITIES

Algebraic Techniques



SKILLS CHECK

1 Determine whether each statement is true or false:

a $3 > 2$

b $-3 > -2$

c $6 < 15$

d $-6 < -15$

e $4 \geq -1$

f $-4 \geq 1$

g $-2 \leq 2$

h $2 \leq -2$

2 Determine whether the following statements are true or false:

a $-3 \geq 2$

b $5 < 2 \times 3$

c $12 + 8 \neq 35 - 5 \times 3$

d $19 > 17 - 3 \times -2$

e $6^2 \leq 2^6$

f $8 + 5 \neq 5 + 8$

3 The given value of the pronumeral satisfies the inequality. True or false?

a $5a - 4 < 2a$ [$a = -2$]

b $\frac{9e-1}{11} \geq 7$ [$e = 5$]

c $3p + 1 \leq 3 - 3p$ [$p = -1$]

d $\frac{x}{3} + 8 \geq -2$ [$x = -30$]

4 Solve:

a $-4p = 8$

b $4p < -8$

c $-4p > 8$

d $3x + 17 = 74$

e $2n - 1 > 15$

f $5x \leq x + 8$

g $\frac{a}{4} + 7 \geq -2$

h $\frac{t-3}{2} > -1$

5 Solve:

a $x + 4 \leq -2$

b $a - 5 > 5$

c $4e < -2$

d $\frac{n}{3} \geq 6$

6 Solve:

a $2x + 5 \geq 11$

b $3a - 1 < -2$

c $\frac{m+2}{3} \leq 4$

7 On separate number lines, graph:

a $x \geq 1$

b $x < 2$

c $x \leq -2$

8 $x = -1$ satisfies the equation $5(x + 1) - 2(3 - 2x) = 10$. True or false?

9 $x = -1$ satisfies the inequality $7x - 3 > x - 8$. True or false?

10 **a** If $A = \frac{1}{2}bh$, find h when $A = 36$ and $b = 8$.

b If the area of a triangle is 84 cm^2 and the perpendicular height is 7 cm , find the length of the base of the triangle.



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Answers 1 a true b false c true d false e true f false g true h false 2 a false b true c false d true e true f false g true h false 3 a true b false c true d true e true f false g true h false 4 a true b false c true d true e true f false g true h false 5 a true b false c true d true e true f false g true h false 6 a true b false c true d true e true f false g true h false 7 a true b false c true d true e true f false g true h false 8 a true b false c true d true e true f false g true h false 9 a true b false c true d true e true f false g true h false 10 a true b false c true d true e true f false g true h false

FURTHER EQUATIONS AND INEQUALITIES

Algebraic Techniques



INTERMEDIATE TEST

Part A Multiple Choice

- Which value of x satisfies the inequality $11 < 2x - 15$?
A $x = -10$ **B** $x = 0$ **C** $x = 9$ **D** none of these (1 mark)
- Which value of m does not satisfy the inequality $\frac{7m}{3} - 8 \leq 9 - \frac{m}{2}$? *Hint 1*
A $m = -2$ **B** $m = 0$ **C** $m = 6$ **D** $m = 9$ (1 mark)
- If $-2x > -6$, which is correct?
A $x > 3$ **B** $x < 3$ **C** $x > -3$ **D** $x < -3$ (1 mark)
- The correct solution to the equation $3p - 17 = 5p + 19$ is: *Hint 2*
A $p = -18$ **B** $p = 18$ **C** $p = -1$ **D** $p = 1$ (1 mark)
- 'When 15 is subtracted from three times a certain number the result is 8 more than twice the number.' Let the number be x . Which statement is correct?
A $3x - 15 = 2x + 8$ **B** $15 - 3x = 2x + 8$ **C** $3x - 15 = 2(x + 8)$ **D** $15 - 3x = 2(x + 8)$ (1 mark)
- The number, 8 more than $2x - 5$ is: *Hint 3*
A $2x + 3$ **B** $2x + 13$ **C** $10x - 5$ **D** $10x + 13$ (1 mark)

Part B Short Answer

- Rikki bought seven bottles of soft drink and a packet of biscuits from the local store. The total of her purchases was \$15.98. If the packet of biscuits cost \$2.75:
a form an equation which could be solved to find the cost of each bottle of soft drink *Hint 4* (1 mark)
b how much would five bottles of the same soft drink cost? *Hint 5* (2 marks)
- Solve $5(2x + 3) \leq 20$ *Hint 6* (1 mark)
- Solve $9n + 4 > 2n - 17$ (2 marks)
- Solve $3 - 2a > 7$, graphing the solution on a number line. *Hint 7* (3 marks)
- Solve $3m + 17 \leq 8m$ (2 marks)
- Solve $2 - \frac{u}{7} \geq 3$ (2 marks)
- 'When 15 is subtracted from a certain number and the result multiplied by 5, the answer is seven more than three times the original number.' Form an equation and solve to find the original number. *Hint 8* (3 marks)
- A particular art competition has special requirements about the size of the entries. Each entry must be presented on a rectangular background, the length of which is to be 20 cm longer than its breadth. The maximum total perimeter allowed is 150 cm. Find the dimensions of the largest possible entry. *Hint 9* (3 marks)

$(x + 20)$ cm
 maximum
 perimeter
 150 cm

x cm

Hint 1: Substitute each answer to see whether it satisfies the inequality.

Hint 2: Either solve the equation or substitute each value to see which one satisfies the equation.

Hint 3: Add 8 to $2x - 5$.

Hint 4: Let x be the price of a bottle of soft drink.

Hint 5: First find the cost of one bottle.

Hint 6: Solve an inequality in the same way as an equation.

Hint 7: If you divide by a negative number, the inequality sign will be reversed.

Hint 8: Let x be the number.

Hint 9: Form an inequality and solve.

Your Feedback

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$\%$



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FURTHER EQUATIONS AND INEQUALITIES

Algebraic Techniques



ADVANCED TEST

1 Graph on a number line:

a $x \geq 3$

b $x < -1$ or $x > 4$

c $-2 \leq x < 5$

(1 mark each)

2 Solve, and graph the solution on a number line:

a $5x + 1 \geq 9 - 3x$

b $3(p + 5) - 1 < 11$

c $9 - 2n > 5$

(2 marks each)

3 Solve:

a $3(2m - 5) \leq 2(m + 7)$

b $\frac{5k}{3} - 2 < 4 - k$

c $\frac{6-a}{2} > 4$

d $7(4q - 1) - 3(2q + 5) \geq 0$

e $12 - \frac{3d}{4} \leq 18$

f $5(2y - 3) - 4(3y - 7) > 2(y + 5)$

(2 marks each)

4 Jo produced the following when trying to solve the equation $x(x + 5) = x(x + 3)$.

$$\begin{array}{r} \frac{x(x+5)}{x} = \frac{x(x+3)}{x} \\ x+5 = x+3 \\ 5 = 3 \end{array}$$

Briefly explain what Jo did wrong and give the correct solution to the equation.

(2 marks)

5 Change the subject of each formula to b :

a $y = mx + b$

(1 mark)

b $A = \frac{1}{2}h(a + b)$

(2 marks)

c $c = \sqrt{a^2 + b^2}$

(2 marks)

6 a Make r the subject of the formula $V = \pi r^2 h$.

(1 mark)

b What restrictions apply to the variables in the formula before and after it is rearranged?

(2 marks)

c Find the value of r to one decimal place ($r > 0$), when $V = 179$ and $h = 8.7$

(1 mark)

7 A right-angled triangle has area 27.3 m^2 and base length 19.5 m .

a What is the perpendicular height of the triangle?

(1 mark)

b Use Pythagoras' theorem to find the length of the hypotenuse.

(1 mark)

c Why do we not consider the negative value when finding the hypotenuse?

(1 mark)

8 a Find the value of $\sqrt{3 - 2x}$ when $x = -23$?

(1 mark)

b What restrictions must be placed on x in $\sqrt{3 - 2x}$?

(1 mark)

c Solve $\sqrt{3 - 2x} = 5$

(1 mark)

d Considering the previous parts of this question, write down the solution to:

i $\sqrt{3 - 2x} > 5$

(1 mark)

ii $\sqrt{3 - 2x} < 5$

(1 mark)

Your Feedback

$$\frac{\boxed{}}{40} \times 100\% = \boxed{}\%$$



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QUADRATIC EQUATIONS

Algebraic Techniques



STUDY NOTES

- 1 A **quadratic equation** is one where the highest power of the unknown is 2. It is an equation involving the square of the unknown (x^2). Some examples of quadratic equations are $x^2 = 9$, $5a^2 = 100$ and $2n^2 - 8n + 13 = 0$.
- 2 Whenever a **negative number** is **squared**, the **answer is positive**. So a simple quadratic equation will have two solutions—the positive square root and the negative square root, e.g. if $x^2 = 16$, $x = 4$ or $x = -4$.
- 3 A **quadratic equation** may have **2, 1 or no solutions**. It will have no solutions if a perfect square is equal to a negative number ($x^2 = -4$ for example). It will have one solution if a perfect square is equal to zero. For example, if $(x - 3)^2 = 0$ then $x = 3$ is the only solution. A quadratic equation will have two solutions if a perfect square is equal to a positive number. For example, if $n^2 = 9$ then $n = 3$ and $n = -3$ are both solutions.
- 4 The **symbol** $\pm$ means ‘plus or minus’ or ‘positive or negative’. If we write $x = \pm 4$ we mean $x = 4$ or $x = -4$.
- 5 **Simple quadratic equations** are solved by taking the **square root** of both sides of the equation, remembering to consider both the **positive and negative** square roots. If $2x^2 = 8$ then $x^2 = 4$ and $x = \pm 2$.
- 6 **Quadratic equations** may be **solved by factorisation**. This method relies on the fact that if two numbers multiply to zero, one (or both) of the numbers must be zero. Using this method, put the expression equal to zero, factorise (if possible), let each of the factorised parts equal zero and solve these simple equations. For example, consider the equation $x^2 - 6x + 8 = 0$. Now, $x^2 - 6x + 8 = (x - 2)(x - 4)$ so we know that $(x - 2)(x - 4) = 0$ and this means that either $x - 2 = 0$ or $x - 4 = 0$. Solving these two simple equations gives $x = 2$ or $x = 4$.
- 7 **Not every quadratic expression can be factorised**. This doesn’t necessarily mean that the equation has no solution.
- 8 The **factorisation method** only works if the **equation is equal to zero**. If $(x - 2)(x - 4) = 8$ say, it doesn’t mean that one part must be 8. You must first expand, put the expression equal to zero and then factorise this new expression.
$$\begin{aligned}(x - 2)(x - 4) &= 8 \\ x^2 - 6x + 8 &= 8 \\ x^2 - 6x &= 0 \\ x(x - 6) &= 0 \\ x = 0 &\text{ or } x = 6\end{aligned}$$
- 9 Like any equations, the **solutions to quadratic equations** can be **checked by substitution**. Take care to substitute the value of the unknown every time it appears in the equation.
- 10 When **answering problems involving the solution of a quadratic**, we should always consider whether both positive and negative answers are **physically possible**. For example, if finding the length of rectangle with a particular area, the length cannot be negative so we only choose the positive answer. Writing ‘but $a > 0$ ’ illustrates that you understand and have considered the negative value.

Checklist

Can you:



- 1 Understand what is meant by a quadratic equation?
- 2 Understand that a quadratic equation may have 0, 1 or 2 solutions?
- 3 Understand that if two numbers multiply to zero, at least one of the numbers must be zero?
- 4 Solve quadratic equations by factorising?
- 5 Check the solutions to quadratic equations?
- 6 Understand that not every quadratic equation can be solved by factorisation?

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QUADRATIC EQUATIONS

Algebraic Techniques

SKILLS CHECK

1 Find the value of x^2 if:

a $x = 3$

b $x = -3$

c $x = 7$

d $x = -7$

e $x = -5$

f $x = 5$

g $x = 1$

h $x = -1$

2 The square of 6 is 36 ($6^2 = 36$).

a The square of what other number is also 36?

b Write down two numbers whose square is 100.

3 Solve:

a $x^2 = 9$

b $x^2 = 49$

c $x^2 = 25$

d $x^2 = 1$

4 Find the value of $2p^2$ if:

a $p = 2$

b $p = -2$

c $p = 6$

d $p = -6$

5 Solve:

a $2p^2 = 8$

b $2p^2 = 72$

6 Solve:

a $x^2 = 16$

b $3e^2 = 243$

c $4k^2 = 1$

7 Solve, giving the answers correct to two decimal places:

a $x^2 = 60$

b $x^2 = 12.5$

c $7x^2 = 45$

8 Solve:

a $5x = 0$

b $3a = 0$

c $12t = 0$

d $-37m = 0$

9 Solve:

a $3(x - 7) = 0$

b $4(2x + 5) = 0$

c $6(3x - 2) = 0$

10 Solve:

a $x(x + 4) = 0$

b $(x - 7)(x + 1) = 0$

c $(2a + 5)(3a - 2) = 0$

11 Factorise:

a $x^2 - 9x$

b $x^2 + 8x + 15$

c $t^2 + 3t - 10$

d $m^2 - 7m + 12$

e $n^2 - 4n - 60$

f $a^2 - 2a + 1$

12 The given value is a solution of the quadratic equation. True or false?

a $x^2 + 7x + 12 = 0$ ($x = 3$)

b $y^2 - 4y - 21 = 0$ ($y = -3$)

13 Solve:

a $n^2 - 3n = 0$

b $x^2 + 6x + 8 = 0$

c $a^2 - 4a - 45 = 0$

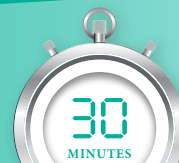


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Answers 1 a 9 b 9 c 49 d 49 e 25 f 25 g 1 h 1 i 2 a 6 b 6 c 6 d 6 e 2 f 2 g 2 h 2 i 2 j 2 k 2 l 2 m 2 n 2 o 2 p 2 q 2 r 2 s 2 t 2 u 2 v 2 w 2 x 2 y 2 z 2 aa 2 ab 2 ac 2 ad 2 ae 2 af 2 ag 2 ah 2 ai 2 aj 2 ak 2 al 2 am 2 an 2 ao 2 ap 2 aq 2 ar 2 as 2 at 2 au 2 av 2 aw 2 ax 2 ay 2 az 2 ba 2 bb 2 bc 2 bd 2 be 2 bf 2 bg 2 bh 2 bi 2 bj 2 bk 2 bl 2 bm 2 bn 2 bo 2 bp 2 bq 2 br 2 bs 2 bt 2 bu 2 bv 2 bw 2 bx 2 by 2 bz 2 ca 2 cb 2 cc 2 cd 2 ce 2 cf 2 cg 2 ch 2 ci 2 cj 2 ck 2 cl 2 cm 2 cn 2 co 2 cp 2 cq 2 cr 2 cs 2 ct 2 cu 2 cv 2 cw 2 cx 2 cy 2 cz 2 da 2 db 2 dc 2 dd 2 de 2 df 2 dg 2 dh 2 di 2 dj 2 dk 2 dl 2 dm 2 dn 2 do 2 dp 2 dq 2 dr 2 ds 2 dt 2 du 2 dv 2 dw 2 dx 2 dy 2 dz 2 ea 2 eb 2 ec 2 ed 2 ee 2 ef 2 eg 2 eh 2 ei 2 ej 2 ek 2 el 2 em 2 en 2 eo 2 ep 2 eq 2 er 2 es 2 et 2 eu 2 ev 2 ew 2 ex 2 ey 2 ez 2 fa 2 fb 2 fc 2 fd 2 fe 2 ff 2 fg 2 fh 2 fi 2 fj 2 fk 2 fl 2 fm 2 fn 2 fo 2 fp 2 fq 2 fr 2 fs 2 ft 2 fu 2 fv 2 fw 2 fx 2 fy 2 fz 2 ga 2 gb 2 gc 2 gd 2 ge 2 gf 2 gg 2 gh 2 gi 2 gj 2 gk 2 gl 2 gm 2 gn 2 go 2 gp 2 gq 2 gr 2 gs 2 gt 2 gu 2 gv 2 gw 2 gx 2 gy 2 gz 2 ha 2 hb 2 hc 2 hd 2 he 2 hf 2 hg 2 hi 2 hj 2 hk 2 hl 2 hm 2 hn 2 ho 2 hp 2 hq 2 hr 2 hs 2 ht 2 hu 2 hv 2 hw 2 hx 2 hy 2 hz 2 ia 2 ib 2 ic 2 id 2 ie 2 if 2 ig 2 ih 2 ii 2 ij 2 ik 2 il 2 im 2 in 2 io 2 ip 2 iq 2 ir 2 is 2 it 2 iu 2 iv 2 iw 2 ix 2 iy 2 iz 2 ja 2 jb 2 jc 2 jd 2 je 2 jf 2 jg 2 jh 2 ji 2 jj 2 jk 2 jl 2 jm 2 jn 2 jo 2 jp 2 jq 2 jr 2 js 2 jt 2 ju 2 jv 2 jw 2 jx 2 jy 2 jz 2 ka 2 kb 2 kc 2 kd 2 ke 2 kf 2 kg 2 kh 2 ki 2 kj 2 kl 2 km 2 kn 2 ko 2 kp 2 kq 2 kr 2 ks 2 kt 2 ku 2 kv 2 kw 2 kx 2 ky 2 kz 2 la 2 lb 2 lc 2 ld 2 le 2 lf 2 lg 2 lh 2 li 2 lj 2 lk 2 ll 2 lm 2 ln 2 lo 2 lp 2 lq 2 lr 2 ls 2 lt 2 lu 2 lv 2 lw 2 lx 2 ly 2 lz 2 ma 2 mb 2 mc 2 md 2 me 2 mf 2 mg 2 mh 2 mi 2 mj 2 mk 2 ml 2 mm 2 mn 2 mo 2 mp 2 mq 2 mr 2 ms 2 mt 2 mu 2 mv 2 mw 2 mx 2 my 2 mz 2 na 2 nb 2 nc 2 nd 2 ne 2 nf 2 ng 2 nh 2 ni 2 nj 2 nk 2 nl 2 nm 2 no 2 np 2 nq 2 nr 2 ns 2 nt 2 nu 2 nv 2 nw 2 nx 2 ny 2 nz 2 oa 2 ob 2 oc 2 od 2 oe 2 of 2 og 2 oh 2 oi 2 oj 2 ok 2 ol 2 om 2 on 2 oo 2 op 2 oq 2 or 2 os 2 ot 2 ou 2 ov 2 ow 2 ox 2 oy 2 oz 2 pa 2 pb 2 pc 2 pd 2 pe 2 pf 2 pg 2 ph 2 pi 2 pj 2 pk 2 pl 2 pm 2 pn 2 po 2 pp 2 pq 2 pr 2 ps 2 pt 2 pu 2 pv 2 pw 2 px 2 py 2 pz 2 qa 2 qb 2 qc 2 qd 2 qe 2 qf 2 qg 2 qh 2 qi 2 qj 2 qk 2 ql 2 qm 2 qn 2 qo 2 qp 2 qq 2 qr 2 qs 2 qt 2 qu 2 qv 2 qw 2 qx 2 qy 2 qz 2 ra 2 rb 2 rc 2 rd 2 re 2 rf 2 rg 2 rh 2 ri 2 rj 2 rk 2 rl 2 rm 2 rn 2 ro 2 rp 2 rq 2 rr 2 rs 2 rt 2 ru 2 rv 2 rw 2 rx 2 ry 2 rz 2 sa 2 sb 2 sc 2 sd 2 se 2 sf 2 sg 2 sh 2 si 2 sj 2 sk 2 sl 2 sm 2 sn 2 so 2 sp 2 sq 2 sr 2 ss 2 st 2 su 2 sv 2 sw 2 sx 2 sy 2 sz 2 ta 2 tb 2 tc 2 td 2 te 2 tf 2 tg 2 th 2 ti 2 tj 2 tk 2 tl 2 tm 2 tn 2 to 2 tp 2 tq 2 tr 2 ts 2 tt 2 tu 2 tv 2 tw 2 tx 2 ty 2 tz 2 ua 2 ub 2 uc 2 ud 2 ue 2 uf 2 ug 2 uh 2 ui 2 uj 2 uk 2 ul 2 um 2 un 2 uo 2 up 2 uq 2 ur 2 us 2 ut 2 uu 2 uv 2 uw 2 ux 2 uy 2 uz 2 va 2 vb 2 vc 2 vd 2 ve 2 vf 2 vg 2 vh 2 vi 2 vj 2 vk 2 vl 2 vm 2 vn 2 vo 2 vp 2 vq 2 vr 2 vs 2 vt 2 vu 2 vv 2 vw 2 vx 2 vy 2 vz 2 wa 2 wb 2 wc 2 wd 2 we 2 wf 2 wg 2 wh 2 wi 2 wj 2 wk 2 wl 2 wm 2 wn 2 wo 2 wp 2 wq 2 wr 2 ws 2 wt 2 wu 2 wv 2 ww 2 wx 2 wy 2 wz 2 xa 2 xb 2 xc 2 xd 2 xe 2 xf 2 xg 2 xh 2 xi 2 xj 2 xk 2 xl 2 xm 2 xn 2 xo 2 xp 2 xq 2 xr 2 xs 2 xt 2 xu 2 xv 2 xw 2 xx 2 xy 2 xz 2 ya 2 yb 2 yc 2 yd 2 ye 2 yf 2 yg 2 yh 2 yi 2 yj 2 yk 2 yl 2 ym 2 yn 2 yo 2 yp 2 yq 2 yr 2 ys 2 yt 2 yu 2 yv 2 yw 2 yx 2 yy 2 yz 2 za 2 zb 2 zc 2 zd 2 ze 2 zf 2 zg 2 zh 2 zi 2 zj 2 zk 2 zl 2 zm 2 zn 2 zo 2 zp 2 zq 2 zr 2 zs 2 zt 2 zu 2 zv 2 zw 2 zx 2 zy 2 zz

QUADRATIC EQUATIONS

Algebraic Techniques



INTERMEDIATE TEST

Part A Multiple Choice

- 1 A solution to the equation $4a^2 = 16$ is: I $a = 2$ II $a = -2$
A I only B II only C neither I nor II D both I and II (1 mark)
- 2 Which is a solution to the equation $p^2 + 8p - 9 = 0$?
A $p = -1$ B $p = 8$ C $p = -8$ D $p = -9$ (1 mark)
- 3 The solutions to the equation $x^2 - 11x + 28 = 0$ are:
A $x = 4$ and $x = 7$ B $x = -4$ and $x = 7$ C $x = 4$ and $x = -7$ D $x = -4$ and $x = -7$ (1 mark)
- 4 A quadratic equation has solutions $x = 3$ and $x = -2$. The equation is:
A $x^2 + 5x - 6 = 0$ B $x^2 - 5x - 6 = 0$ C $x^2 + x - 6 = 0$ D $x^2 - x - 6 = 0$ (1 mark)
- 5 The solution to the equation $(x - 5)(x - 2) = 28$ is: *Hint 1*
A $x = 2$ or $x = 5$ B $x = 7$ or $x = 4$ C $x = -2$ or $x = 9$ D $x = -5$ or $x = 3$ (1 mark)

Part B Short Answer

- 6 Solve $100x^2 = 1$ *Hint 2* (2 marks)
- 7 Solve $4x^2 - 100 = 0$ *Hint 3* (2 marks)
- 8 Solve by first factorising:
a $x^2 + 11x = 0$ *Hint 4* (2 marks)
b $x^2 + 2x - 80 = 0$ (2 marks)
c $x^2 - 18x + 81 = 0$ (2 marks)
d $x^2 + 14x + 48 = 0$ (2 marks)
- 9 Solve $(2x + 3)^2 = 25$ *Hint 5* (2 marks)
- 10 a A class was asked to solve the equation $(x + 3)(x + 5) = 35$. One student said that because the product is 35 one of the factors must be 35 so the answer was $x = 32$ or $x = 30$. Briefly explain what is wrong with this student's logic. (2 marks)
b Solve $(x + 3)(x + 5) = 35$ *Hint 6* (3 marks)
- 11 A rectangle is 12 metres longer than it is wide. The area of the rectangle is 45 m^2 .
a Let the width of the rectangle be $x \text{ m}$. Explain why the width of the rectangle can be found by solving the quadratic equation $x^2 + 12x - 45 = 0$. (3 marks)
b Find the dimensions of the rectangle. *Hint 7* (3 marks)

Hint 1: Check by substitution.

Hint 2: There are two answers.

Hint 3: Add 100 to both sides and then divide both sides by 4.

Hint 4: Take out a common factor.

Hint 5: Take the square root of both sides of the equation. There will be two answers.

Hint 6: Expand the binomial first and then put the equation equal to zero.

Hint 7: Make sure your answers are physically possible.

Your Feedback

$\times 100\% =$ %



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QUADRATIC EQUATIONS

Algebraic Techniques



ADVANCED TEST

1 Solve:

a $a^2 = \frac{1}{16}$

(1 mark)

b $4t^2 = 25$

(1 mark)

c $12d^2 - 27 = 0$

(1 mark)

2 Explain why there is no real solution to the equation $x^2 + 1 = 0$.

(1 mark)

3 Solve:

a $x^2 + 9x + 14 = 0$

(1 mark)

b $a^2 - 2a - 48 = 0$

(1 mark)

c $54 - 3x - x^2 = 0$

(1 mark)

4 Solve:

a $x^2 = 5x$

(2 marks)

b $n^2 = 12n + 64$

(2 marks)

c $s^2 + 5s = 24$

(2 marks)

5 Solve, giving each answer correct to two decimal places:

a $x^2 = 7$

(2 marks)

b $3y^2 = 11$

(2 marks)

c $5m^2 - 13 = 0$

(2 marks)

6 Solve:

a $2x^2 - 28x + 66 = 0$

(2 marks)

b $3m^2 + 30m - 72 = 0$

(2 marks)

c $5y^2 - 5y - 10 = 0$

(2 marks)

7 Solve:

a $y^3 + 7y^2 = 0$

(2 marks)

b $a^3 - 9a = 0$

(2 marks)

c $x^4 - 5x^2 + 4 = 0$

(2 marks)

8 Solve:

a $(x - 5)(x - 2) = -2$

(2 marks)

b $(x + 6)(x - 7) = x + 21$

(2 marks)

c $2x^2 = (x - 5)(x - 12)$

(2 marks)

9 Solve $(x + 4)(x + 6) + (x + 8)(x - 3) = (x - 12)(x + 2)$

(3 marks)

Your Feedback

$\times 100\% =$ %



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SIMULTANEOUS EQUATIONS

Algebraic Techniques



STUDY NOTES

- 1 **Simultaneous equations** are equations that are solved together.
- 2 The **simultaneous solution** of two equations in two unknowns (variables), is the values of those variables which **satisfy both equations** at the **same time**. For example, the simultaneous solution of $y = x + 1$ and $y = 2x$ is $x = 1$ and $y = 2$.
- 3 Two equations are independent if they are different equations. ($x + y = 4$ and $3x + 3y = 12$ are not independent because the second equation is just three times the first equation.) Whenever the equations are independent, the **solution to two simultaneous linear equations is unique**. There are an infinite number of solutions that satisfy each equation separately but only one solution that satisfies both equations at the same time.
- 4 The **solution of two linear simultaneous equations** can be found by **graphing** each line on a **number plane**, and finding the **point of intersection** of the two graphs. (The disadvantages of this method include the time needed to draw the graphs and the difficulty to find the solution with any accuracy unless the answer involves exact integral values.)
- 5 The **solution to two simultaneous equations** can be found by **substituting** one equation into the other. This method is particularly appropriate if one of the unknowns is the subject of one of the equations. For example, to solve $3x + 2y - 11 = 0$ and $y = 4x$ simultaneously, substitute $y = 4x$ into $3x + 2y - 11 = 0$, giving $3x + 2(4x) - 11 = 0$. This new equation can easily be solved to give $x = 1$. Substituting this value for x into either of the original equations gives $y = 4$.
- 6 **Simultaneous equations** can also be **solved by an elimination method**. Under this method the equations are added together, or one equation is subtracted from the other, so that one of the unknowns is eliminated. For example, if $3x + y = 8$ and $5x + y = 10$ then subtracting the first equation from the second gives $2x = 2$, so $x = 1$ and, upon substituting $x = 1$ into either equation, $y = 5$.
- 7 If the **coefficients of an unknown are equal**, and of the same sign (both positive or both negative) then **subtracting** one equation from the other will eliminate that unknown. It doesn't matter whether the first equation is subtracted from the second or vice versa. If the coefficients of an unknown are equal but of different signs, then **adding** the two equations together will eliminate the unknown.
- 8 A **variable** will only be eliminated if the **coefficients are the same**. It may be necessary to multiply either or both equations so that two coefficients are equal. For example, to solve $2x + 3y - 5 = 0$ and $x + 6y - 8 = 0$ simultaneously, multiply the first equation by 2 so that 6 is the coefficient of y in both equations. (Alternatively, the first equation could be multiplied by 5, and the second equation by 2, so that the coefficients of x are equal.) Care must be taken to **multiply every term** in the equation.
- 9 Whatever method is used to solve simultaneous equations, the **solution can always be checked** by substitution. If the solution satisfies both equations, it must be correct.
- 10 It is also possible to solve **three simultaneous equations** in three unknowns by first using two of the equations to eliminate one variable and then another two equations (one used before, one not) to eliminate the same variable and then solving these two new equations simultaneously.

Checklist

Can you:

- 1 Understand what is meant by simultaneous equations?
- 2 Solve simultaneous equations graphically?
- 3 Solve simultaneous equations by substituting one equation into another?
- 4 Solve simultaneous equations by the elimination method?

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SIMULTANEOUS EQUATIONS

Algebraic Techniques



SKILLS CHECK

1 Solve:

a $3a + 11 = -1$

b $4x + 3(6x - 5) - 2 = 16$

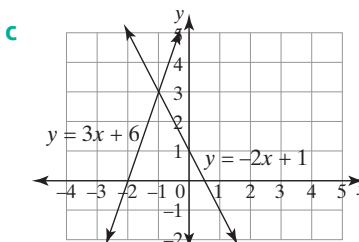
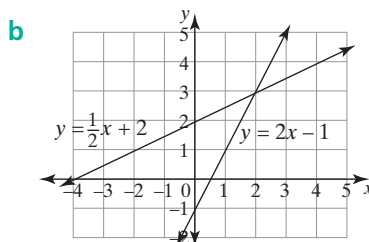
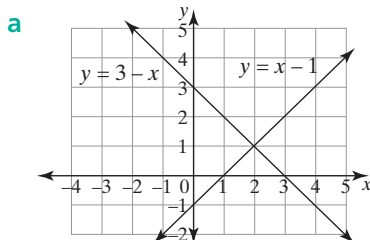
2 If $q = -2$, find p when:

a $p + q = 5$

b $3p - q = 17$

c $4p + 3q - 10 = 0$

3 Find the values of x and y at the point where the two lines intersect.



4 Tables of values have been drawn up for the equations $y = 2x - 5$ and $y = x + 1$. By considering the patterns, find the values of x and y which satisfy both equations.

$y = 2x - 5$

x	1	2	3	4
y	-3	-1	1	3

$y = x + 1$

x	1	2	3	4
y	2	3	4	5

5 a Draw up a table of values, for $x = 0, 1, 2, 3, 4, 5$ and 6 , and complete for the rule $y = 3x - 8$.

b For what values in the table does $x = y$?

c For what values in the table does $2x + y = 7$?

6 Make y the subject of each equation:

a $7x - y = 10$

b $5x + y - 2 = 0$

c $6x + 2y - 1 = 0$

7 Write down the new equation formed when equation [i] is added to equation [ii]. (There is no need to solve.)

a $2a + b = 5$ [i]
 $3a - b = 5$ [ii]

b $8x + 5y - 2 = 0$ [i]
 $3x - 5y + 13 = 0$ [ii]

c $7p - 3q = 15$ [i]
 $2p + 3q = 18$ [ii]

8 Subtract equation [ii] from equation [i]:

a $6x + 7y = 9$ [i]
 $2x + 7y = 13$ [ii]

b $11a - 3b = 8$ [i]
 $5a - 3b = 2$ [ii]

c $5a - 3b - 1 = 0$ [i]
 $5a + 6b + 10 = 0$ [ii]

9 Determine whether adding both equations or subtracting one equation from the other will eliminate a variable.

a $7a - 4b = 9$ [i]
 $3a + 4b = 31$ [ii]

b $8x + 2y = 19$ [i]
 $8x - 2y = 15$ [ii]

c $4p - 3q + 7 = 0$ [i]
 $p - 3q - 15 = 0$ [ii]

10 A box holds only red and blue pens. Let x be the number of blue pens and y the number of red pens. Write algebraic statements to represent the given information.

a There are 15 pens in the box.

b There are five more blue pens than red pens.

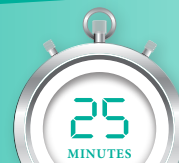
c There are twice as many blue pens as red pens.



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SIMULTANEOUS EQUATIONS

Algebraic Techniques



INTERMEDIATE TEST

Part A Multiple Choice

- 1 At what point do the lines $x = 3$ and $y = 4$ intersect?
A (3, 4) **B** (4, 3) **C** (-3, -4) **D** (-4, -3) (1 mark)
- 2 The simultaneous solution of the equations $2x + y = 5$ and $x - 3y = 13$ is: *Hint 1*
A $x = 4, y = 3$ **B** $x = 4, y = -3$ **C** $x = -4, y = 3$ **D** $x = -4, y = -3$ (1 mark)
- 3 For what value of y do the equations $y = 4x + 7$ and $y = -2x - 5$ have a simultaneous solution?
A -1 **B** -2 **C** 1 **D** 2 (1 mark)
- 4 Which two equations have a simultaneous solution where $x = 0$?
A $y = 3 - \frac{1}{2}x$ and $y = x - 6$ **B** $y = -\frac{1}{2}x - 6$ and $y = 3 - \frac{1}{2}x$
C $y = x - 6$ and $y = -\frac{1}{2}x - 6$ **D** $y = x + 3$ and $y = -\frac{1}{2}x - 6$ (1 mark)

Part B Short Answer

- 5 **a** Draw a set of axes and graph the lines $y = x + 3$ and $y = 7 - x$. (2 marks)
b Write down the simultaneous solution of the two equations in part **a**. *Hint 2* (2 marks)
- 6 Solve the simultaneous equations $7x - 2y = 5$ and $y = 3x$ by substitution. *Hint 3* (2 marks)
- 7 Solve by elimination $7a + 2b = 16$ and $3a - 2b = 24$. *Hint 4* (2 marks)
- 8 **a** Make y the subject of the equation $2x + y - 5 = 0$. *Hint 5* (1 mark)
b Solve the equations $2x + y - 5 = 0$ and $3x - 4y + 42 = 0$ by substitution. (2 marks)
- 9 Solve by elimination $9m + 4p = 41$ and $5m + 2p = 23$. *Hint 6* (2 marks)
- 10 Solve simultaneously: $2x - y + 8 = 0$ and $3x - y - 2 = 0$. *Hint 7* (2 marks)
- 11 Find the simultaneous solution of the equations $5a + 3b = 8$ and $3a - 4b = 28$. (3 marks)
- 12 Madeline bought a total of 24 cans of dog food and cat food at her local store. The dog food cost \$1.40 per can and the cat food was 90c per can. Altogether Madeline spent \$29.10 on the pet food. How many cans of each type of food did she buy? *Hint 8* (4 marks)

Hint 1: There is no need to solve the equations simultaneously. Substitute to see which solution satisfies both equations.

Hint 2: What are the coordinates of the point of intersection?

Hint 3: Replace y with $3x$ in the first equation.

Hint 4: Add the two equations together so that b is eliminated.

Hint 5: The subject of an equation is the value before the equals sign. Rearrange the equation so that it reads $y = \dots$

Hint 6: First multiply the second equation by 2. Make sure that you multiply every term in the equation.

Hint 7: It doesn't matter which method you use.

Hint 8: Let x be the number of cans of dog food and y be the number of cans of pet food. Form two different equations from the given information.

Your Feedback

$$\frac{\boxed{}}{26} \times 100\% = \boxed{}\%$$



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SIMULTANEOUS EQUATIONS

Algebraic Techniques



ADVANCED TEST

- 1 Use the substitution method to solve simultaneously:
 - a $y = 5x - 6$ and $y = 3x + 2$ (2 marks)
 - b $y = 7 - 2x$ and $x + 3y = 11$ (2 marks)
 - c $3p - 5q = 23$ and $4p + q = 23$ (2 marks)

- 2 Use the elimination method to solve simultaneously:
 - a $a + b = 4$ and $a - b = -10$ (2 marks)
 - b $8x - 3y = 30$ and $5x - 3y = 3$ (2 marks)
 - c $9m + 5n - 3 = 0$ and $9m - 7n + 69 = 0$ (2 marks)
 - d $4p + 3q = 27$ and $3p + q = 14$ (2 marks)
 - e $9a - 4b = -1$ and $5a + 2b = -9$ (2 marks)
 - f $6k - 5m = 19$ and $7k - 3m = 42$ (2 marks)

- 3 a Graph the lines $y = 2x - 4$, $y = -2x + 8$, $y = \frac{1}{2}x - 1$, $y = 5 - x$ and $y = -3$ on the same set of axes. (3 marks)
- b Write down the simultaneous solution of $y = \frac{1}{2}x - 1$ and:
 - i $y = 2x - 4$ (1 mark)
 - ii $y = 5 - x$ (1 mark)
 - iii $y = -3$ (1 mark)
- c What is significant about the point $(3, 2)$? (1 mark)

- 4 a For what value(s) of x does $x = x^2$? (1 mark)
- b What is the simultaneous solution of $y = x$ and $y = x^2$? (1 mark)
- c In how many places will the line $y = x$ intersect with the parabola $y = x^2$? (1 mark)
- d In how many places will the line $y = 4$ intersect with the parabola $y = x^2$? (1 mark)
- e What is the simultaneous solution of $y = 4$ and $y = x^2$? (1 mark)

- 5 Solve simultaneously:
 - a $y = x^2 + 5x$ and $y = x + 12$ (3 marks)
 - b $y = x^2 + x - 10$ and $y = 2x - 4$ (3 marks)

- 6 Solve $y = x^2 + 8x + 3$ and $y = x^2 - 2x - 7$ simultaneously. (2 marks)

- 7 a Give a reason why a graphical means of solving two equations simultaneously might be the best option. (1 mark)
- b Give a reason why an algebraic means of solving two equations simultaneously might be the best option. (1 mark)

Your Feedback

$$\frac{\boxed{}}{40} \times 100\% = \boxed{}\%$$



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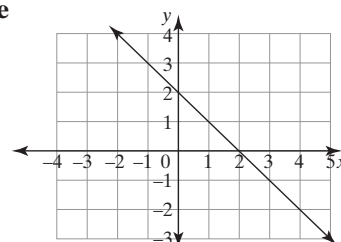
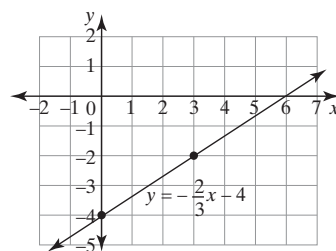
PARALLEL AND PERPENDICULAR LINES

Linear and Non-linear Relationships



STUDY NOTES

- The **gradient** (or slope) of a line is the measure of its steepness. A line with **positive gradient** leans to the right (/) and a line with **negative gradient** leans to the left (\). The symbol **m** is used for gradient. The gradient is given by $m = \frac{\text{rise}}{\text{run}}$ or the **gradient formula**: $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- The **y-intercept** is where a line meets the y-axis. It is the **value of y when $x = 0$** . The symbol **b** is used for the y-intercept.
- Any **equation of the form $y = mx + b$** is the **equation of a straight line**.
- In the equation $y = mx + b$, **m** , the coefficient of x (the number by which x is multiplied) gives the **gradient**. For example, the line $y = 3x + 4$ has gradient 3. The line $y = -x + 2$ has gradient -1 .
- The constant term, **b** , gives the **y-intercept**. For example, The line $y = 3x + 4$ has y-intercept 4. The line $y = 2x - 1$ has y-intercept -1 . The line $y = -x$ has y-intercept 0. (It passes through the origin $(0, 0)$.)
- We say $y = mx + b$ is the **gradient/intercept form** of the equation of a straight line.
- Given the gradient and y-intercept, any **line can be graphed**. The y-intercept gives us one point on the line. Use the gradient, $\frac{\text{rise}}{\text{run}}$, to find a second point. For example, if $y = \frac{2}{3}x - 4$, the y-intercept is -4 so one point on the line is $(0, -4)$. The gradient is $\frac{2}{3}$ so run across 3 units and rise up 2 units to find another point on the line.
- The gradient and y-intercept can be **read from a graph**. The **equation of the line** can then be formed. For example, the gradient of this line is -1 . The y-intercept is 2. So the equation of the line is $y = -x + 2$ ($y = -1x + 2$).
- Two lines are parallel** if their gradients are equal. For two lines with gradients m_1 and m_2 , if $m_1 = m_2$ the lines are parallel and if the lines are parallel $m_1 = m_2$. To show that two lines are parallel show that their gradients are equal.
- Two lines are perpendicular** if they meet at right angles. If two lines are perpendicular their gradients are negative reciprocals. For two lines with gradients m_1 and m_2 , the lines are perpendicular if $m_1 m_2 = -1$, and if $m_1 m_2 = -1$ the lines are perpendicular. (Note: if $m_1 m_2 = -1$ then $m_2 = \frac{-1}{m_1}$). To show that two lines are perpendicular, show that their gradients multiply to give negative one.



Checklist

Can you:

- Understand what is meant by gradient?
- Understand what is meant by the y-intercept?
- Understand that $y = mx + b$ is the equation of a straight line in gradient/intercept form?
- Understand that in the equation $y = mx + b$, m is the gradient and b is the y-intercept?
- Graph lines using the gradient and y-intercept?
- Find the gradient, y-intercept and equation of a line from a graph?
- Understand that parallel lines have the same gradient?
- Understand what it means for two lines to be perpendicular?
- Understand that the gradients of perpendicular lines are negative reciprocals?


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PARALLEL AND PERPENDICULAR LINES

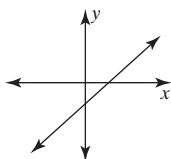
Linear and Non-linear Relationships



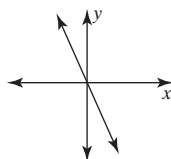
SKILLS CHECK

- 1 State whether the gradient of each line is positive or negative:

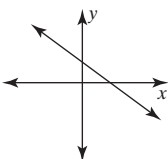
a



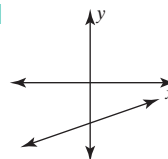
b



c

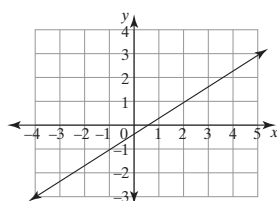


d

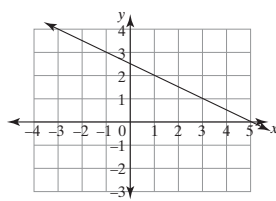


- 2 Find the gradient of each of these lines:

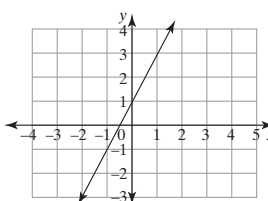
a



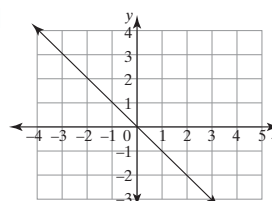
b



c



d



- 3 Find the gradient of the line joining (1, 2) to (5, 0).

- 4 Write down the gradient (m) and y-intercept (b) for each line:

a $y = 2x + 3$

b $y = 4x - 1$

c $y = -6x$

d $y = x - 5$

e $y = \frac{1}{3}x - 2$

f $y = \frac{x}{4}$

g $y = \frac{-2x}{3} + \frac{1}{2}$

h $y = 5 - 2x$

- 5 Write down the equation of the line with gradient m and y-intercept b if:

a $m = 3, b = 2$

b $m = -2, b = 0$

c $m = \frac{1}{2}, b = -\frac{1}{3}$

- 6 Complete:

a If the gradients of two lines are equal then the lines are _____.

b If two lines are parallel then their gradients are _____.

- 7 The gradients of two lines are m_1 and m_2 . Complete the following statements.

a If $m_1 = m_2$ the two lines are _____.

b If $m_1 m_2 = -1$ the two lines are _____.

- 8 A line l has gradient $-\frac{1}{2}$. Find the gradient of any line which is:

a parallel to l

b perpendicular to l

- 9 The two lines are parallel. True or false?

a $y = 3x - 5$

b $y = \frac{1}{2}x - 4$

c $y = 7 - x$

$y = 3x + 8$

$y = \frac{x}{2} + 7$

$y = -x + 2$

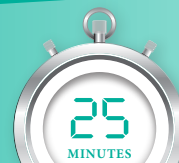


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Answers 1 a positive b negative c negative d positive 2 a $\frac{3}{2}$ b $-\frac{1}{2}$ c 2 d -1 e $-\frac{1}{3}$ f $-\frac{2}{3}$ g $-\frac{1}{2}$ h $-\frac{3}{2}$ i $-\frac{1}{2}$ j $-\frac{1}{2}$ k $-\frac{1}{2}$ l $-\frac{1}{2}$ m $-\frac{1}{2}$ n $-\frac{1}{2}$ o $-\frac{1}{2}$ p $-\frac{1}{2}$ q $-\frac{1}{2}$ r $-\frac{1}{2}$ s $-\frac{1}{2}$ t $-\frac{1}{2}$ u $-\frac{1}{2}$ v $-\frac{1}{2}$ w $-\frac{1}{2}$ x $-\frac{1}{2}$ y $-\frac{1}{2}$ z $-\frac{1}{2}$ 7 a parallel b perpendicular 8 a $-\frac{1}{2}$ b 2 9 a true b true c true

PARALLEL AND PERPENDICULAR LINES

Linear and Non-linear Relationships



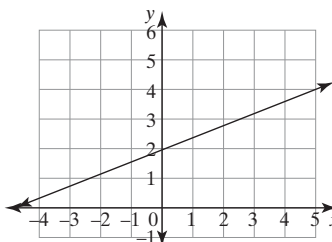
INTERMEDIATE TEST

Part A Multiple Choice

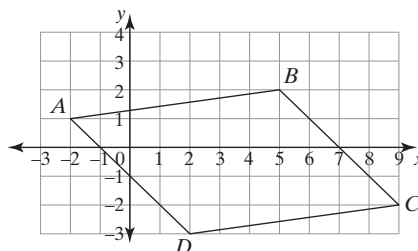
- 1 Which line is parallel to $y = 3x - 4$?
A $y = 2x - 4$ **B** $y = -3x + 4$ **C** $y = \frac{-1}{3}x - 4$ **D** $y = 3x + 7$ (1 mark)
- 2 Any line perpendicular to the line $y = 2x - 5$ has gradient:
A 2 **B** -2 **C** $\frac{1}{2}$ **D** $-\frac{1}{2}$ (1 mark)
- 3 The line which is parallel to the line $y = 2 - x$ and which passes through the origin has equation:
A $y = 2x$ **B** $y = -x$ **C** $x = 2$ **D** $x + y = 2$ (1 mark)

Part B Short Answer

- 4 Consider the line shown in the diagram.
a What is the gradient of the line? (1 mark)
b What is the y-intercept? (1 mark)
c Copy the diagram, and then draw the line $y = \frac{2}{5}x + 4$. *Hint 1* (2 marks)



- 5 A , B , C and D are the points $(-2, 1)$, $(5, 2)$, $(9, -2)$ and $(2, -3)$ respectively. They form the vertices of a quadrilateral.
a Show that AB is parallel to DC . *Hint 2* (2 marks)
b Show that AD is parallel to BC . (2 marks)
c What type of quadrilateral is $ABCD$? (1 mark)



- 6 $P(-8, 5)$, $Q(10, 6)$ and $R(-2, -3)$ are the vertices of a triangle. Is $\triangle PQR$ a right-angled triangle? Justify your answer. *Hint 3* (4 marks)
- 7 The equation of line l is $y = -4x + 3$.
a Line k is parallel to line l and has y-intercept -1 . What is the equation of line k ? (2 marks)
b Line m is perpendicular to line l and meets it on the y-axis. What is the equation of line m ? *Hint 4* (3 marks)
- 8 A is the point $(7, -8)$, B is the point $(1, 1)$ and C is the point $(-3, 7)$.
a Find the gradient of AB . (1 mark)
b Find the gradient of BC . (1 mark)
c What conclusion can be made about points A , B and C ? Justify your answer. (2 marks)

Hint 1: What do you know about the two lines?

Hint 2: Find the gradient of both lines.

Hint 3: Draw a diagram and then investigate whether any sides are perpendicular.

Hint 4: Lines l and m have the same y-intercept.

Your Feedback

$$\frac{\boxed{}}{25} \times 100\% = \boxed{}\%$$



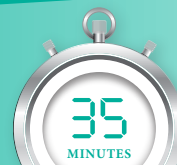
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PARALLEL AND PERPENDICULAR LINES

Linear and Non-linear Relationships



ADVANCED TEST

- 1 Determine whether each pair of lines is parallel, perpendicular or neither:
 - a $y = 3x - 2$; $x + 3y - 4 = 0$ (2 marks)
 - b $2x + y - 3 = 0$; $x + 2y + 5 = 0$ (2 marks)
 - c $y = \frac{3}{4}x - 5$; $3x - 4y - 2 = 0$ (2 marks)

- 2 Line l has equation $y = 7 - x$. What is the equation of the line:
 - a parallel to l that passes through $(2, 1)$? (2 marks)
 - b perpendicular to line l that intersects it on the y -axis? (2 marks)

- 3 P is the point $(1, 8)$ and Q is the point $(-4, 5)$.
 - a Find the gradient of the line joining P and Q . (1 mark)
 - b R is the point on the x -axis such that $PQ \perp QR$. What are the coordinates of R ? (2 marks)

- 4 $S(-10, -5)$, $T(-1, 7)$ and $U(15, -5)$ are the vertices of a triangle.
 - a Find the gradient of ST . (1 mark)
 - b Show that $\triangle STU$ is right-angled at T . (2 marks)
 - c Write down the coordinates of V such that $STUV$ is a parallelogram. (1 mark)
 - d Find the area of $STUV$. (2 marks)

- 5 $A(-2, 4)$, $B(6, 2)$ and $C(-4, 2)$ are vertices of a triangle.
 - a What is the midpoint of AB ? (1 mark)
 - b What is the gradient of AB ? (1 mark)
 - c What is the equation of the perpendicular bisector of AB ? (1 mark)
 - d What is the equation of the perpendicular bisector of AC ? (2 marks)
 - e What is the point of intersection of the lines found in parts c and d? (2 marks)
 - f What is the equation of the perpendicular bisector of BC ? (1 mark)
 - g What do you notice about the three perpendicular bisectors? (1 mark)

- 6
 - a Rearrange the equation $ax + by + c = 0$ into gradient/intercept form. (1 mark)
 - b What is the gradient of any line parallel to $ax + by + c = 0$? (1 mark)
 - c What is the gradient of any line parallel to $5x - 3y + 7 = 0$? (1 mark)
 - d Write down the equation, in general form, of the line through the origin parallel to the line $5x - 3y + 7 = 0$? (1 mark)
 - e What is the gradient of any line perpendicular to $ax + by + c = 0$? (1 mark)
 - f What is the gradient of any line perpendicular to $4x + 2y - 3 = 0$? (1 mark)
 - g Write down, in general form, the equation of the line through the origin perpendicular to $4x + 2y - 3 = 0$ (1 mark)

Your Feedback

$$\frac{\boxed{}}{35} \times 100\% = \boxed{}\%$$



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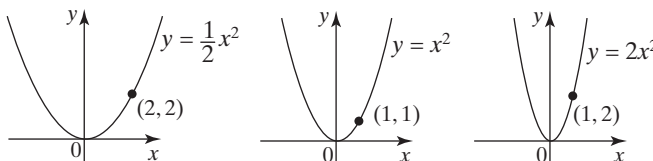
FURTHER GRAPHS

Linear and Non-linear Relationships



STUDY NOTES

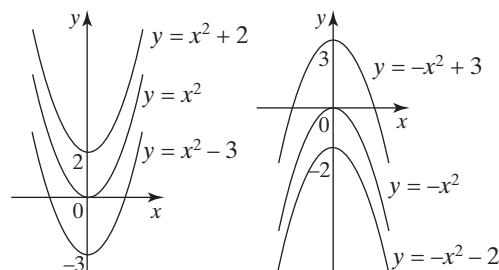
- 1 The **equation** $y = mx + b$ is the **gradient/intercept form** of an equation of a **straight line** (with gradient m and y -intercept b). Any equation of the form $Ax + By + C = 0$ is the equation of a straight line in general form. These equations can be changed from one to the other.



- 2 A **parabola** is a special type of curve. The graph of $y = ax^2 + c$ ($a \neq 0$) is a parabola. If $a > 0$, the parabola opens upwards ($\cup$) but if $a < 0$ the parabola opens downwards ($\cap$).

- 3 The **turning point** of a parabola is called the **vertex**.

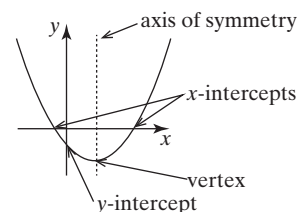
- 4 The parabola $y = ax^2$ has its **vertex at the origin**. The value of a determines the width of the parabola. The larger the value of a , the thinner the parabola.



- 5 The **constant, c**, in the equation $y = ax^2 + c$ has the effect of moving the parabola up or down the y -axis. The vertex of the parabola $y = ax^2 + c$ is at the point $(0, c)$.

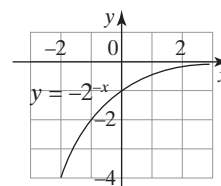
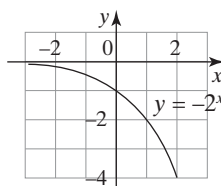
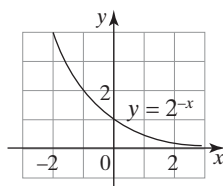
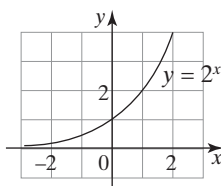
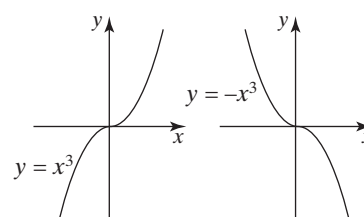
- 6 The graph of $y = ax^2 + bx + c$ (where a, b and c are constants and $a \neq 0$) is also a **parabola**.

- 7 The **x-intercepts** of a parabola are the points where the curve meets the x -axis. A parabola may meet the x -axis in 0, 1 or 2 points. The x -intercepts are found when $y = 0$. The y -intercept is the point where the curve meets the y -axis. The **y-intercept** occurs when $x = 0$. If the equation of the parabola is of the form $y = ax^2 + bx + c$ then the y -intercept will be c .



- 8 The graph of $y = ax^3 + c$ is a **cubic curve**. The value of a determines the width of the curve. c gives the y -intercept.

- 9 Any equation of the form $y = a^x$ is an **exponential curve**. $y = a^x$ has y -intercept 1 for all positive values of a .



- 10 A **circle, centre the origin**, and **radius** of length r units has equation $x^2 + y^2 = r^2$.

Checklist

Can you:

- 1 Identify the type of graph from an equation?
- 2 Graph parabolas, cubic curves, exponential curves and circles?
- 3 Find the radius of a circle (centre the origin) given its equation?
- 4 Use technology (such as graphics calculators or software applications) to draw graphs?



FURTHER GRAPHS

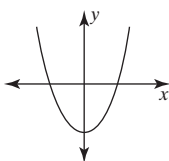
Linear and Non-linear Relationships



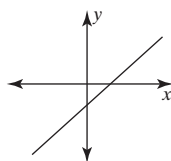
SKILLS CHECK

1 What name is given to each of these curves?

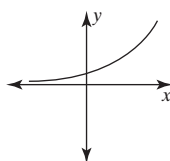
a



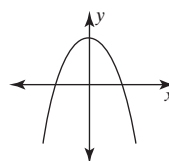
b



c



d



2 Use the parabola shown to answer the following questions.

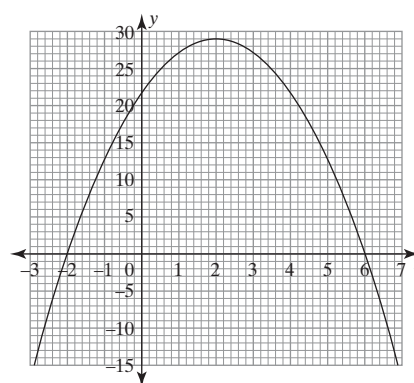
a What is the y-intercept?

b What is the value of y when $x = 4$?

c At what points does the parabola cut the x -axis?

d What are the coordinates of the vertex (turning point)?

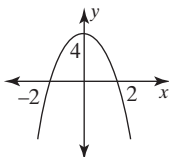
e For what values of x does $y = 13$?



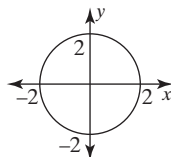
3 Choose the correct equation from the following list to match each curve.

$$y = x^2 - 2, \quad y = -2^x, \quad y = 4 - x^2, \quad y = 4^x, \quad x^2 + y^2 = 4, \quad y = -2x, \quad y = 2^{-x}, \quad y = \frac{x}{4}$$

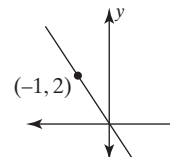
a



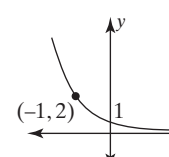
b



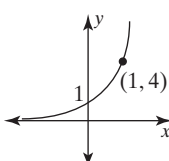
c



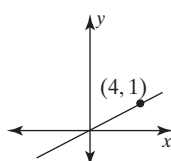
d



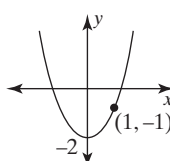
e



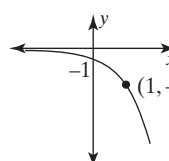
f



g



h



4 Name the type of graph (straight line, parabola, circle, exponential or cubic curve):

a $y = 3 - x$

b $y = 3x^2$

c $y = 3^x$

d $y = x^3$

e $y = x^2 + 3$

f $x^2 + y^2 = 3$

g $y = \frac{x}{3}$

h $y = -x^2 + 2$

i $y = 9 - x^2$

j $x + y = 9$

k $x^2 + y^2 = 9$

l $y = 2^{-x}$

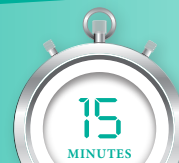


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Answers 1 a parabola b straight line c exponential curve d parabola e parabola f circle g straight line h parabola i parabola j straight line
2 a y-intercept is 20 b y = 20 c (-2, -10) and (6, -10) d (2, 30) e x = -2 and x = 6
3 a y = 3 - x b y = 3x^2 c y = 3^x d y = x^3 e y = x^2 + 3 f x^2 + y^2 = 3 g y = x/3 h y = -x^2 + 2 i y = 9 - x^2 j x + y = 9 k x^2 + y^2 = 9 l y = 2^{-x}

FURTHER GRAPHS

Linear and Non-linear Relationships



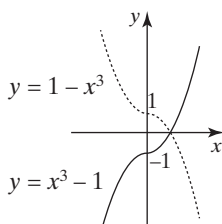
INTERMEDIATE TEST

Part A Multiple Choice

- Which is not the equation of a straight line?
A $x + y = 3$ **B** $x - y = 0$
C $y = \frac{x+3}{2}$ **D** $y = \frac{3}{x}$ (1 mark)
- Which point does not lie on the circle $x^2 + y^2 = 50$?
A $(-1, -7)$ **B** $(7, -1)$
C $(2, 6)$ **D** $(5, 5)$ (1 mark)
- Which diagram could show the graph of $y = x^2 + x$? *Hint 1*
A **B**
C **D** (1 mark)
- Which is the equation of a parabola?
A $y = (x+1)(x-1)$ **B** $y = 2^x$
C $y = x^4$ **D** $y = \frac{4}{x^2}$ (1 mark)

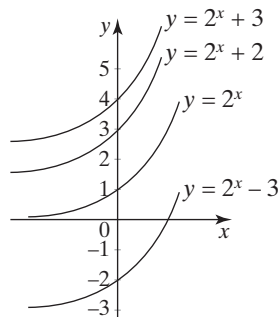
Part B Short Answer

- Kevin used a software program to graph $y = x^3 - 1$ and $y = 1 - x^3$ on the same diagram.

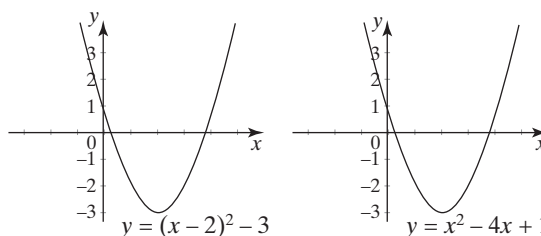


- How many solutions are there to the equation $x^3 - 1 = 1 - x^3$? *Hint 2* (1 mark)
 - Solve $x^3 - 1 = 1 - x^3$. (1 mark)
- A point was plotted on a grid using a software application. The distance from the point $O(0, 0)$ to P was given as 8 units. The circle $x^2 + y^2 = 36$ will then be drawn on the same grid. Will the point P lie inside, outside or on the circle? Justify your answer. (2 marks)

- A graphics calculator was used to show some exponential graphs. All of the graphs had an equation of the form $y = 2^x + c$ where c is a constant.



- What effect does the constant have on the graphs? *Hint 3* (1 mark)
 - What would the y-intercept be of the graph $y = 2^x + 7$? (1 mark)
 - For what value of c will $y = 2^x + c$ pass through the origin? *Hint 4* (1 mark)
- Stacy used a graphics calculator to graph $y = (x-2)^2 - 3$ and $y = x^2 - 4x + 1$



- What similarities can you notice with the two graphs? (1 mark)
- Use algebra to explain the result in part a. *Hint 5* (3 marks)

Hint 1: Substitute some of the points that each parabola passes through.

Hint 2: Use the graph.

Hint 3: What happens to the graphs as the number added changes?

Hint 4: The origin is the point $(0, 0)$

Hint 5: Expand and simplify $(x-2)^2 - 3$.

Your Feedback

$$\frac{\boxed{}}{15} \times 100\% = \boxed{}\%$$



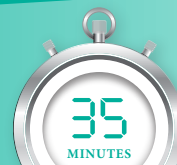
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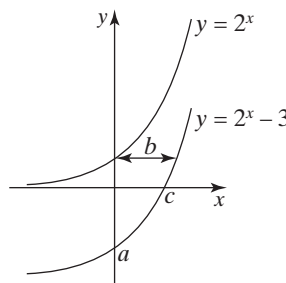
FURTHER GRAPHS

Linear and Non-linear Relationships



ADVANCED TEST

- 1 Consider the curve $y = x^2 - 10x + 21$.
 - a What name is given to the type of curve? (1 mark)
 - b What is the y -intercept? (1 mark)
 - c What are the x -intercepts? (2 marks)
 - d What is the equation of the axis of symmetry? (1 mark)
 - e What is the minimum value? (1 mark)
 - f Sketch the curve. (3 marks)
- 2 Briefly explain any similarities and differences between the curve $y = x^2$ and each of the following:
 - a $y = x^2 + 2$ and $y = x^2 - 3$ (1 mark)
 - b $y = (x + 2)^2$ and $y = (x - 3)^2$ (1 mark)
 - c $y = -x^2$ (1 mark)
 - d $y = 2x^2$ and $y = \frac{1}{2}x^2$ (1 mark)
- 3 Find the turning point and state whether it is a maximum or minimum:
 - a $y = (x + 2)^2 - 3$ (2 marks)
 - b $y = 5 - (3 - x)^2$ (2 marks)
- 4 Describe the curve given by the equation $(x - 1)^2 + (y + 3)^2 = 16$. (2 marks)
- 5 Describe the symmetrical properties of the curve $y = x^n$ if:
 - a n is even (1 mark)
 - b n is odd (1 mark)
- 6 Consider the curve $y = (x - 2)(x - 4)(x + 1)$.
 - a Where will the curve cross the y -axis? (1 mark)
 - b Where will the curve meet the x -axis? (1 mark)
 - c If x has a large positive value, what can you say about the value of y ? (1 mark)
 - d If x has a large negative value, what can you say about the value of y ? (1 mark)
 - e Using the information found, sketch the curve $y = (x - 2)(x - 4)(x + 1)$. (3 marks)
- 7
 - a Solve simultaneously $x^2 + y^2 = 25$ and $y = 7 - x$. (2 marks)
 - b Sketch $x^2 + y^2 = 25$ and $y = 7 - x$ on the same diagram, clearly showing the points of intersection. (2 marks)
- 8 The diagram shows the graphs of $y = 2^x$ and $y = 2^x - 3$.
 - a What is the value of a ? (1 mark)
 - b Find the value of b . (1 mark)
 - c Use your calculator to find the value of c to three decimal places. (1 mark)



Your Feedback

/ 35 × 100% = %



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SPECIAL BINOMIAL PRODUCTS AND FACTORISATIONS

SKILLS CHECK

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$$\begin{aligned} 1 \quad a \quad (x+3)(x+8) \\ = x^2 + 8x + 3x + 24 \\ = x^2 + 11x + 24 \end{aligned}$$

$$\begin{aligned} b \quad (a-5)(a+9) \\ = a^2 + 9a - 5a - 45 \\ = a^2 + 4a - 45 \end{aligned}$$

$$\begin{aligned} c \quad (2a+3)(5b+4) \\ = 10ab + 8a + 15b + 12 \end{aligned}$$

$$\begin{aligned} d \quad (6x-1)(x-8) \\ = 6x^2 - 48x - x + 8 \\ = 6x^2 - 49x + 8 \end{aligned}$$

$$\begin{aligned} e \quad (5n+3)(2n-3) \\ = 10n^2 - 15n + 6n - 9 \\ = 10n^2 - 9n - 9 \end{aligned}$$

$$\begin{aligned} f \quad (x+y)(a+b) \\ = ax + bx + ay + by \end{aligned}$$

$$\begin{aligned} 2 \quad a \quad (x+3)^2 \\ = x^2 + 2 \times x \times 3 + 3^2 \\ = x^2 + 6x + 9 \end{aligned}$$

$$\begin{aligned} b \quad (x-6)^2 \\ = x^2 - 2 \times x \times 6 + 6^2 \\ = x^2 - 12x + 36 \end{aligned}$$

$$\begin{aligned} c \quad (3x+2)^2 \\ = (3x)^2 + 2 \times 3x \times 2 + 2^2 \\ = 9x^2 + 12x + 4 \end{aligned}$$

$$\begin{aligned} d \quad (2x-5)^2 &= 4x^2 - 20x + 25 \\ [(2x)^2] \end{aligned}$$

$$\begin{aligned} 3 \quad a \quad (x+4)^2 &= x^2 + 2 \times x \times 4 + 4^2 \\ &= x^2 + 8x + 16 \end{aligned}$$

$$\begin{aligned} b \quad (x-9)^2 &= x^2 - 18x + 81 \\ [x^2 - 2 \times x \times 9 + 9^2] \end{aligned}$$

$$\begin{aligned} c \quad (3a-10)^2 \\ = (3a)^2 - 2 \times 3a \times 10 + 10^2 \\ = 9a^2 - 60a + 100 \end{aligned}$$

$$\begin{aligned} d \quad (2p+3q)^2 \\ = 4p^2 + 12pq + 9q^2 \\ [(2p)^2 + 2 \times 2p \times 3q + (3q)^2] \end{aligned}$$

$$\begin{aligned} e \quad (7-x)^2 \\ = 7^2 - 2 \times 7 \times x + x^2 \\ = 49 - 14x + x^2 \end{aligned}$$

$$\begin{aligned} f \quad (a+b)^2 \\ = a^2 + 2 \times a \times b + b^2 \\ = a^2 + 2ab + b^2 \end{aligned}$$

$$\begin{aligned} 4 \quad a \quad (p+3)(p-3) &= p^2 - 3^2 \\ &= p^2 - 9 \end{aligned}$$

$$\begin{aligned} b \quad (2x-1)(2x+1) &= (2x)^2 - 1^2 \\ &= 4x^2 - 1 \end{aligned}$$

$$\begin{aligned} c \quad (3x+2y)(3x-2y) \\ = (3x)^2 - (2y)^2 \\ = 9x^2 - 4y^2 \end{aligned}$$

$$\begin{aligned} d \quad (x^2+2)(x^2-2) &= (x^2)^2 - 2^2 \\ &= x^4 - 4 \end{aligned}$$

$$\begin{aligned} 5 \quad a \quad (x-9)(x+9) &= x^2 - 9^2 \\ &= x^2 - 81 \end{aligned}$$

$$\begin{aligned} b \quad (3x-1)(3x+1) &= (3x)^2 - 1^2 \\ &= 9x^2 - 1 \end{aligned}$$

$$\begin{aligned} c \quad (2n+7)(2n-7) &= (2n)^2 - 7^2 \\ &= 4n^2 - 49 \end{aligned}$$

$$d \quad (8-x)(8+x) = 64 - x^2$$

$$\begin{aligned} e \quad (7x+2y)(7x-2y) \\ = (7x)^2 - (2y)^2 \\ = 49x^2 - 4y^2 \end{aligned}$$

$$\begin{aligned} f \quad (ab+2)(ab-2) &= (ab)^2 - 2^2 \\ &= a^2b^2 - 4 \end{aligned}$$

$$\begin{aligned} 6 \quad a \quad x^2 + 13x + 42 &= (x+6)(x+7) \\ [6+7=13, 6 \times 7=42] \end{aligned}$$

$$\begin{aligned} b \quad x^2 + 3x - 40 &= (x+8)(x-5) \\ [\text{Or } (x-5)(x+8)] \end{aligned}$$

$$\begin{aligned} c \quad x^2 - 10x + 16 \\ = (x-2)(x-8) \end{aligned}$$

$$d \quad x^2 - 3x - 54 = (x+6)(x-9)$$

$$7 \quad a \quad x^2 + 8x + 16 = (x+4)^2$$

$$b \quad a^2 - 10a + 25 = (a-5)^2$$

$$c \quad n^2 + 2n + 1 = (n+1)^2$$

$$d \quad x^2 - 16x + 64 = (x-8)^2$$

$$8 \quad a \quad x^2 + 20x + 100 = (x+10)^2$$

$$b \quad y^2 + 4y + 4 = (y+2)^2$$

$$c \quad 49 + 14n + n^2 = (7+n)^2$$

$$d \quad a^2 - 2a + 1 = (a-1)^2$$

$$e \quad 9 - 6m + m^2 = (3-m)^2$$

$$f \quad x^2 - 2xy + y^2 = (x-y)^2$$

$$9 \quad a \quad x^2 - 49 = (x+7)(x-7)$$

$$b \quad a^2 - 9 = (a+3)(a-3)$$

$$c \quad 25 - m^2 = (5+m)(5-m)$$

$$d \quad x^2 - y^2 = (x+y)(x-y)$$

$$\begin{aligned} 10 \quad a \quad x^2 - 81 &= x^2 - 9^2 \\ &= (x+9)(x-9) \\ [\text{Or } (x-9)(x+9)] \end{aligned}$$

$$\begin{aligned} b \quad 121 - y^2 &= 11^2 - y^2 \\ &= (11+y)(11-y) \end{aligned}$$

WORKED SOLUTIONS



CHECK YOUR SOLUTIONS

$$\text{c } m^2 - 900 = m^2 - 30^2 \\ = (m + 30)(m - 30)$$

$$\text{d } p^2 - q^2 = (p + q)(p - q)$$

$$\text{11 a } x^2 + xy + 2x + 2y \\ = x(x + y) + 2(x + y) \\ = (x + y)(x + 2)$$

$$\text{b } ab + 3a - 5b - 15 \\ = a(b + 3) - 5(b + 3) \\ = (b + 3)(a - 5)$$

$$\text{c } a^2 - ab - ac + bc \\ = a(a - b) - c(a - b) \\ = (a - b)(a - c)$$

$$\text{d } 2mn - 2m + 3n - 3 \\ = 2m(n - 1) + 3(n - 1) \\ = (n - 1)(2m + 3)$$

$$\text{e } xz - yz - x + y \\ = z(x - y) - 1(x - y) \\ = (x - y)(z - 1)$$

[It is not necessary to write the 1 in the 2nd line but it can't be left out in the last line.]

$$\text{f } 6kn - 3k - 2n + 1 \\ = 3k(2n - 1) - 1(2n - 1) \\ = (2n - 1)(3k - 1)$$

SPECIAL BINOMIAL PRODUCTS AND FACTORISATIONS

INTERMEDIATE TEST PAGE 27

$$\text{1 } (2x - 5)^2 \\ = (2x)^2 - 2 \times 2x \times 5 + 5^2 \\ = 4x^2 - 20x + 25 \\ \therefore [\text{C}] \quad \checkmark \quad (1 \text{ mark})$$

$$\text{2 } (3x - y)(3x + y) \\ = (3x)^2 - y^2 \\ = 9x^2 - y^2 \\ \therefore [\text{C}] \quad \checkmark \quad (1 \text{ mark})$$

$$\text{3 } xy - 4x - 6y + 24 \\ = x(y - 4) - 6(y - 4) \\ = (y - 4)(x - 6) \\ \therefore [\text{A}] \quad \checkmark \quad (1 \text{ mark})$$

$$\text{4 } x^2 + 34x + 289 = (x + 17)^2 \\ \therefore [\text{B}] \quad \checkmark \quad (1 \text{ mark})$$

$$\text{5 } 6k(k + 2) - (k + 2) \\ = 6k(k + 2) - 1(k + 2) \\ = (k + 2)(6k - 1) \\ \therefore [\text{D}] \quad \checkmark \quad (1 \text{ mark})$$

$$\text{6 } \frac{x^2 - 64}{8x - 64} = \frac{(x + 8)(x - 8)}{8(x - 8)} \\ = \frac{x + 8}{8}$$

[After cancelling the common factor $(x - 8)$, the fraction $\frac{x + 8}{8}$ cannot be simplified.]

8 cannot be cancelled because it is not a common factor.]

$\therefore [\text{C}] \quad \checkmark \quad (1 \text{ mark})$

$$\text{7 } \text{Try each option.}$$

$$x^2 - 49 = (x + 7)(x - 7) \\ \text{not } (x - 7)^2$$

$$x^2 - 10x - 24 = (x - 12)(x + 2) \\ \text{not } (x - 4)(x - 6)$$

$$x^2 - 24x + 144 = (x - 12)^2 \\ \text{not } (x + 12)^2$$

$$x^2 - 625 = (x + 25)(x - 25) \\ \therefore [\text{D}] \quad \checkmark \quad (1 \text{ mark})$$

$$\text{8 } \text{Try each option.}$$

$$x^2 + 40x + 400 = (x + 20)^2$$

$$x^2 - 38x + 361 = (x - 19)^2$$

$$x^2 + 42x + 441 = (x + 21)^2$$

$x^2 - 44x + 464$ is not a perfect square.

$\therefore [\text{D}] \quad \checkmark \quad (1 \text{ mark})$

$$\text{9 } (3x + 5)^2 - (3x - 5)^2 \\ = (3x + 5 + 3x - 5) \\ \times (3x + 5 - (3x - 5)) \\ = 6x \times 10 \\ = 60x \\ \therefore [\text{D}] \quad \checkmark \quad (1 \text{ mark})$$

$$\text{10 } (3x + 8)^2 + (x - 8)(x + 8) \\ = 9x^2 + 48x + 64 + x^2 - 64 \quad \checkmark \\ = 10x^2 + 48x \quad \checkmark \quad (2 \text{ marks})$$

$$\text{11 } a^2 + ab - a - b \\ = a(a + b) - 1(a + b) \quad \checkmark \\ = (a + b)(a - 1) \quad \checkmark \quad (2 \text{ marks})$$

$$\text{12 } a^2b[(2a + b)^2 - (2a - b)^2] \\ = a^2b[(4a^2 + 4ab + b^2) \\ - (4a^2 - 4ab + b^2)] \\ = a^2b[4a^2 + 4ab + b^2 - 4a^2 \\ + 4ab - b^2] \quad \checkmark \\ = a^2b[8ab] \\ = 8a^3b^2 \quad \checkmark \quad (2 \text{ marks})$$

$$\text{13 } (2a - 1)^2 \\ = (2a)^2 - 2 \times 2a \times 1 + 1^2 \\ = 4a^2 - 4a + 1 \quad \checkmark \quad (1 \text{ mark})$$

$$\text{14 a } 7p^2 - 63 \\ = 7(p^2 - 9) \quad \checkmark \\ = 7(p + 3)(p - 3) \quad \checkmark \quad (2 \text{ marks})$$

$$\text{b } 6x^2 - 48x + 96 \\ = 6(x^2 - 8x + 16) \quad \checkmark \\ = 6(x - 4)^2 \quad \checkmark \quad (2 \text{ marks})$$

$$\text{15 a i } \text{When } x = 2, \\ 2x(x + 3) - (x + 2)(x - 2) \\ = 2 \times 2 \times (2 + 3) \\ - (2 + 2) \times (2 - 2) \\ = 20 - 0 - 25 \\ = -5 \quad \checkmark \quad (1 \text{ mark})$$

$$\text{ii } \text{When } x = 4, \\ 2x(x + 3) - (x + 2)(x - 2) \\ = 2 \times 4 \times (4 + 3) \\ - (4 + 2) \times (4 - 2) \\ = 56 - 12 - 49 \\ = -5 \quad \checkmark \quad (1 \text{ mark})$$

$$\text{iii } \text{When } x = -1, \\ 2x(x + 3) - (x + 2)(x - 2) \\ = 2(-1)(-1 + 3) \\ - (-1 + 2)(-1 - 2) \\ = -4 + 3 - 4 \\ = -5 \quad \checkmark \quad (1 \text{ mark})$$

$$\text{b } 2x(x + 3) - (x + 2)(x - 2) \\ = 2x^2 + 6x - (x^2 - 4) \\ = 2x^2 + 6x - x^2 + 4 - x^2 \\ = -5 \\ \therefore 2x(x + 3) - (x + 2)(x - 2) \\ = (x + 3)^2 \\ \text{will always equal } -5. \quad \checkmark \quad (2 \text{ marks})$$

(Total: 25 marks)

SPECIAL BINOMIAL PRODUCTS AND FACTORISATIONS

ADVANCED TEST PAGE 28

$$\text{1 a } (4p + 5q)^2 \\ = (4p)^2 + 2 \times 4p \times 5q + (5q)^2 \\ = 16p^2 + 40pq + 25q^2 \quad \checkmark \quad (1 \text{ mark})$$

WORKED SOLUTIONS



CHECK YOUR SOLUTIONS

$$\begin{aligned} \text{b } (12-p)(12+p) \\ = 12^2 - p^2 \\ = 144 - p^2 \quad \checkmark \quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{c } (5x-3y)(5x+3y) \\ = (5x)^2 - (3y)^2 \\ = 25x^2 - 9y^2 \quad \checkmark \quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{d } 2(x-4)^2 \\ = 2(x^2 - 8x + 16) \\ = 2x^2 - 16x + 32 \quad \checkmark \\ (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{e } 3x(x+3)(x-3) \\ = 3x(x^2 - 9) \\ = 3x^3 - 27x \quad \checkmark \quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{f } (4x^2y^3 - 7z)(4x^2y^3 + 7z) \\ = (4x^2y^3)^2 - (7z)^2 \\ = 16x^4y^6 - 49z^2 \quad \checkmark \\ (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} 2 \text{ a } 81 - 4d^2 &= (9 - 2d)(9 + 2d) \quad \checkmark \\ &\quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{b } 36 - 12c + c^2 &= (6 - c)^2 \quad \checkmark \\ &\quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{c } 4n^2 - 12n + 9 \\ = (2n - 3)^2 \quad \checkmark \quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{d } x^4 - 25y^2 \\ = (x^2 - 5y)(x^2 + 5y) \quad \checkmark \\ (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{e } x^4 + 4x^2 + 4 \\ = (x^2 + 2)^2 \quad \checkmark \quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{f } p^2 - p + \frac{1}{4} &= \left(p - \frac{1}{2}\right)^2 \quad \checkmark \\ &\quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} 3 \text{ a } 2x^2 - 18 \\ = 2(x^2 - 9) \\ = 2(x+3)(x-3) \quad \checkmark \\ (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{b } 3x^2 + 30x + 75 \\ = 3(x^2 + 10x + 25) \\ = 3(x+5)^2 \quad \checkmark \quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{c } x^4 - 1 \\ = (x^2 + 1)(x^2 - 1) \\ = (x^2 + 1)(x+1)(x-1) \quad \checkmark \\ (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{d } p - p^3 \\ = p(1 - p^2) \\ = p(1 + p)(1 - p) \quad \checkmark \\ (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{e } 2x^3 - 12x^2 + 18x \\ = 2x(x^2 - 6x + 9) \\ = 2x(x-3)^2 \quad \checkmark \quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{f } 16a^4 - 72a^2 + 81 \\ = (4a^2 - 9)^2 \\ = [(2a+3)(2a-3)]^2 \quad \checkmark \\ (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} 4 \text{ a } (5x-2)(7-x) + (3x-2)^2 \\ = 35x - 5x^2 - 14 + 2x \\ + 9x^2 - 12x + 4 \quad \checkmark \\ = 4x^2 + 25x - 10 \quad \checkmark \\ (2 \text{ marks}) \end{aligned}$$

$$\begin{aligned} \text{b } (x+y)(x-y) - (x-y)^2 \\ = x^2 - y^2 - (x^2 - 2xy + y^2) \quad \checkmark \\ = x^2 - y^2 - x^2 + 2xy - y^2 \\ = 2xy - 2y^2 \quad \checkmark \quad (2 \text{ marks}) \end{aligned}$$

$$\begin{aligned} \text{c } 9(m+5)(m-2) \\ - (3m-10)(3m+10) \\ = 9(m^2 + 3m - 10) \\ - (9m^2 - 100) \quad \checkmark \\ = 9m^2 + 27m - 90 - 9m^2 \\ + 100 \\ = 27m + 10 \quad \checkmark \quad (2 \text{ marks}) \end{aligned}$$

$$\begin{aligned} \text{d } (5a-2b)^2 - (5a+2b)^2 \\ = 25a^2 - 20ab + 4b^2 \\ - (25a^2 + 20ab + 4b^2) \quad \checkmark \\ = 25a^2 - 20ab + 4b^2 - 25a^2 \\ - 20ab - 4b^2 \\ = -40ab \quad \checkmark \quad (2 \text{ marks}) \end{aligned}$$

$$\begin{aligned} 5 \text{ a } \frac{9p^2 + 9}{9p^2 + 18p + 9} \\ = \frac{9(p^2 + 1)}{9(p^2 + 2p + 1)} \quad \checkmark \\ = \frac{p^2 + 1}{(p+1)^2} \quad \checkmark \quad (2 \text{ marks}) \end{aligned}$$

$$\begin{aligned} \text{b } \frac{n^2 - 100}{n^2 - 12n + 20} \\ = \frac{(n+10)(n-10)}{(n-10)(n-2)} \quad \checkmark \\ = \frac{n+10}{n-2} \quad \checkmark \quad (2 \text{ marks}) \end{aligned}$$

$$\begin{aligned} \text{c } \frac{x^3 - 4x}{x^3 - 4x^2 + 4x} \\ = \frac{x(x^2 - 4)}{x(x^2 - 4x + 4)} \\ = \frac{(x+2)(x-2)}{(x-2)^2} \quad \checkmark \\ = \frac{x+2}{x-2} \quad \checkmark \quad (2 \text{ marks}) \end{aligned}$$

$$\begin{aligned} \text{d } \frac{x^2 - 2x + 1}{1 - x^2} \\ = \frac{(x-1)^2}{(1+x)(1-x)} \quad \checkmark \\ = \frac{-(x-1)}{1+x} \\ = \frac{1-x}{1+x} \quad \checkmark \quad (2 \text{ marks}) \end{aligned}$$

$$\begin{aligned} 6 \text{ a } \frac{1}{x^2 + 20x + 100} - \frac{1}{x^2 - 100} \\ = \frac{1}{(x+10)^2} - \frac{1}{(x+10)(x-10)} \quad \checkmark \\ = \frac{(x-10) - (x+10)}{(x+10)^2(x-10)} \\ = \frac{-20}{(x+10)^2(x-10)} \quad \checkmark \\ (2 \text{ marks}) \end{aligned}$$

$$\begin{aligned} \text{b } \frac{1}{a^3 - 1} \div \frac{1}{a^6 - 1} \\ = \frac{1}{a^3 - 1} \times \frac{a^6 - 1}{1} \quad \checkmark \\ = \frac{(a^3 - 1)(a^3 + 1)}{a^3 - 1} \\ = a^3 + 1 \quad \checkmark \quad (2 \text{ marks}) \end{aligned}$$

$$\begin{aligned} 7 \text{ a } 1636 &= 1637 - 1 \\ \text{and } 1638 &= 1637 + 1 \\ \text{So } 1636 \times 1638 \\ &= (1637 - 1)(1637 + 1) \\ &= 1637^2 - 1 \\ &= 2679769 - 1 \\ &= 2679768 \quad \checkmark \quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{b } 897 \times 903 \\ = (900 - 3)(900 + 3) \\ = 900^2 - 3^2 \\ = 810000 - 9 \\ = 809991 \quad \checkmark \quad (1 \text{ mark}) \\ \text{(Total: 40 marks)} \end{aligned}$$

SUBSTITUTION, EQUATIONS AND FORMULAE SKILLS CHECK

PAGE 30

- 1 If $x = 4$,
 - a $x + 3 = 4 + 3 = 7$
 - b $x - 6 = 4 - 6 = -2$
 - c $9 - x = 9 - 4 = 5$
 - d $5x = 5 \times 4 = 20$
 - e $2x + 1 = 2 \times 4 + 1 = 9$
 - f $3x - 2 = 3 \times 4 - 2 = 10$
 - g $x^2 = 4^2 = 16$
 - h $2x^2 = 2 \times 4^2 = 2 \times 16 = 32$

WORKED SOLUTIONS



CHECK YOUR SOLUTIONS

2 If $a = 2, b = 5, c = 6$,

a $a + b = 2 + 5$
 $= 7$

b $c - b = 6 - 5$
 $= 1$

c $b + c - a = 5 + 6 - 2$
 $= 9$

d $3a = 3 \times 2$
 $= 6$

e $bc = 5 \times 6$
 $= 30$

f $ab + c = 2 \times 5 + 6$
 $= 16$

g $3ac = 3 \times 2 \times 6$
 $= 36$

h $a^2 + b^2 = 2^2 + 5^2$
 $= 4 + 25$
 $= 29$

3 a $x + 13 = 25$
 $x = 25 - 13$
 $x = 12$

b $y - 9 = 37$
 $y = 37 + 9$
 $y = 46$

c $5a = 325$
 $a = 325 \div 5$
 $a = 65$

d $\frac{h}{6} = 12$
 $h = 12 \times 6$
 $h = 72$

e $2x + 1 = 17$ [subtract 1]
 $2x = 16$ [divide by 2]
 $x = 8$

f $3x - 4 = 5$ [add 4]
 $3x = 9$ [divide by 3]
 $x = 3$

g $\frac{k}{2} + 3 = 7$ [subtract 3]
 $\frac{k}{2} = 4$ [multiply by 2]
 $k = 8$

h $\frac{d-4}{3} = -9$ [multiply by 3]
 $d-4 = -27$ [add 4]
 $d = -23$

4 a $5x + 8 = 29 - 2x$
Substitute $x = 3$,
LHS $= 5 \times 3 + 8$
 $= 23$

RHS $= 29 - 2 \times 3$
 $= 23$

LHS = RHS True

b $\frac{x}{3} + \frac{x}{4} = 7$

Substitute $x = 12$ in LHS

$\frac{12}{3} + \frac{12}{4} = 4 + 3$
 $= 7$

LHS = RHS True

c $4(2x + 1) - 3(x + 5) = 28$

Substitute $x = 8$ in LHS

$4(2 \times 8 + 1) - 3(8 + 5)$
 $= 4 \times 17 - 3 \times 13$
 $= 29$

LHS $\neq$ RHS False

d $\frac{7x+1}{5} = \frac{2x-9}{3}$

Substitute $x = -4$

LHS $= \frac{7 \times -4 + 1}{5}$
 $= -5\frac{2}{5}$

RHS $= \frac{2 \times -4 - 9}{3}$
 $= -5\frac{2}{3}$

LHS $\neq$ RHS False

5 a $9p = 15 - p$
[add p to both sides]
 $10p = 15$
[divide both sides by 10]
 $p = 1.5$

b $7k + 5 = 2k + 25$
[subtract $2k$ from both sides]
 $5k + 5 = 25$
 $5k = 20$
 $k = 4$

6 $A = 120, h = 12.5$
 $V = Ah$
 $= 120 \times 12.5$
 $= 1500$

7 $l = 8, = 5$
 $P = 2l + 2b$
 $= 2 \times 8 + 2 \times 5$
 $= 26$

8 $u = 28, a = -4, t = 5$
 $v = u + at$
 $= 28 + (-4) \times 5$
 $= 28 - 20$
 $= 8$

9 $a = 24, b = 45$
 $c^2 = a^2 + b^2$
 $= 24^2 + 45^2$
 $= 2601$
 $c = \sqrt{2601} \quad (c > 0)$
 $= 51$

10 $r = 12$
 $A = \pi r^2$
 $= \pi \times 12^2$
 $= 452.3893421 \dots$
 $= 452.4 \quad (1 \text{ d.p.})$

11 $x_1 = -2, x_2 = 1, y_1 = 8, y_2 = -4$
 $m = \frac{y_2 - y_1}{x_2 - x_1}$
 $= \frac{-4 - 8}{1 - (-2)}$
 $= -4$

SUBSTITUTION, EQUATIONS AND FORMULAE

INTERMEDIATE TEST PAGE 31

1 $a = 5, b = -3$
 $3a - b = 3 \times 5 - (-3)$
 $= 15 + 3$
 $= 18$
 $\therefore [B] \checkmark \quad (1 \text{ mark})$

2 $2(3x + 4) = 2x - 4$
 $6x + 8 = 2x - 4$
 $4x + 8 = -4$
 $4x = -12$
 $x = -3$
[Or try each option to see which one satisfies the equation.]
 $\therefore [B] \checkmark \quad (1 \text{ mark})$

3 If $x = -2$,
 $3x^2 = 3 \times (-2)^2$
 $= 3 \times 4$
 $= 12$
 $\therefore [C] \checkmark \quad (1 \text{ mark})$

4 $A = (n - 2) \times 180^\circ$
When $n = 15$,
 $A = (15 - 2) \times 180^\circ$
 $= 2340^\circ$
 $\therefore [D] \checkmark \quad (1 \text{ mark})$

5 $y = mx + b$
When $m = 2, b = -6$ and $y = 8$
 $8 = 2x - 6$
 $14 = 2x$
 $7 = x$
 $\therefore [D] \checkmark \quad (1 \text{ mark})$

WORKED SOLUTIONS



CHECK YOUR SOLUTIONS

6 $v = u + at$
 $u = v - at$ (A is not correct)
 $v = u + at$
 $at = v - u$
 $a = \frac{v - u}{t}$ (B is correct)
 $t = \frac{v - u}{a}$
 (neither C nor D is correct)
 $\therefore$ [B] ✓ (1 mark)

7 $6a + 9 - a = 8 - 3a + 1$
 $5a + 9 = 9 - 3a$ ✓
 $8a + 9 = 9$
 $8a = 0$ ✓
 $a = 0$ ✓ (3 marks)

8 $4(x - 5) + 3(x - 2) = 16$
 $4x - 20 + 3x - 6 = 16$ ✓
 $7x - 26 = 16$ ✓
 $7x = 42$
 $x = 6$ ✓ (3 marks)

9 $\frac{2t + 4}{7} - 3 = 5$
 $\frac{2t + 4}{7} = 8$ ✓
 $2t + 4 = 56$ ✓
 $2t = 52$
 $t = 26$ ✓
 [Or multiply each term by 7 first;
 $2t + 4 - 21 = 35$
 $2t - 17 = 35$
 $2t = 52$
 $t = 26$] (3 marks)

10 $\frac{2m - 5}{3} = \frac{3m + 2}{5}$
 $5(2m - 5) = 3(3m + 2)$ ✓
 $10m - 25 = 9m + 6$ ✓
 $m - 25 = 6$
 $m = 31$ ✓ (3 marks)

11 a Between lines 1 and 2 ✓
 because $3x - 5x = -2x$
 (not $8x$) ✓
 Between lines 3 and 4 ✓
 because $x = 4 \div 8$
 (not $8 \div 4$) ✓ (4 marks)

b $3x + 5 - 5x = 9$
 $-2x + 5 = 9$
 $-2x = 4$
 $x = -2$ ✓ (1 mark)

12 $A = 20, y = 8$
 $D = \frac{yA}{y + 12}$

$= \frac{8 \times 20}{8 + 12}$ ✓
 $= \frac{160}{20}$
 $= 8$
 The child's dose is 8 mL. ✓
 (2 marks)
 (Total: 25 marks)

SUBSTITUTION, EQUATIONS AND FORMULAE

ADVANCED TEST

PAGE 32

1 a $5(x - 3) + 2(x - 7) = 6$
 $5x - 15 + 2x - 14 = 6$ ✓
 $7x - 29 = 6$
 $7x = 35$
 $x = 5$ ✓ (2 marks)

b $4(3k - 2) = 3(1 - 2k)$
 $12k - 8 = 3 - 6k$ ✓
 $18k = 11$
 $k = \frac{11}{18}$ ✓ (2 marks)

c $3(4p - 5) = 2p - (7 - p)$
 $12p - 15 = 2p - 7 + p$ ✓
 $9p = 8$
 $p = \frac{8}{9}$ ✓ (2 marks)

d $11(7 - 2g) - 5(3g - 8) = 8g$
 $77 - 22g - 15g + 40 = 8g$ ✓
 $117 = 45g$
 $g = 2.6$ ✓ (2 marks)

e $\frac{3a}{4} + 7 = 2a + 1$
 $3a + 28 = 8a + 4$ ✓
 $24 = 5a$
 $a = 4.8$ ✓ (2 marks)

f $\frac{9n - 3}{7} = 4n + 5$
 $9n - 3 = 28n + 35$ ✓
 $-19n = 38$
 $n = -2$ ✓ (2 marks)

g $\frac{5x - 3}{4} = \frac{4x - 2}{3}$
 $3(5x - 3) = 4(4x - 2)$
 $15x - 9 = 16x - 8$ ✓
 $-x = 1$
 $x = -1$ ✓ (2 marks)

h $\frac{m}{5} + \frac{m}{3} = 16$
 $3m + 5m = 240$ ✓
 $8m = 240$
 $m = 30$ ✓ (2 marks)

i $\frac{3u + 2}{5} - \frac{4u - 1}{2} = \frac{2}{3}$
 $30\left(\frac{3u + 2}{5}\right) - 30\left(\frac{4u - 1}{2}\right) = 30 \times \frac{2}{3}$
 $6(3u + 2) - 15(4u - 1) = 20$
 $18u + 12 - 60u + 15 = 20$ ✓
 $-42u = -7$
 $u = \frac{1}{6}$ ✓ (2 marks)

2 a $x^2 = 0.25$
 $x = \pm 0.5$ ✓ (1 mark)

b $9a^2 = 1$
 $a^2 = \frac{1}{9}$
 $a = \pm \frac{1}{3}$ ✓ (1 mark)

c $t^2 - 4 = 0$
 $t^2 = 4$
 $t = \pm 2$ ✓ (1 mark)

d $\frac{y^2}{2} - 5 = 3$
 $\frac{y^2}{2} = 8$
 $y^2 = 16$
 $y = \pm 4$ ✓ (1 mark)

3 a $5x^2 - 2 = 7$
 $5x^2 = 9$
 $x^2 = 1.8$
 $x = \sqrt{1.8}$ ($x > 0$) ✓
 $= 1.341\ 6407\ldots$
 $= 1.342$ (3 d.p.) ✓ (2 marks)

b It means there is only one answer instead of two. ✓ (1 mark)

4 a $5p - 3pq$
 $= 5 \times -2 - 3 \times -2 \times -3$
 $= -28$ ✓ (1 mark)

b $4p^2 = 4 \times (-2)^2$
 $= 16$ ✓ (1 mark)

c $\frac{p + q}{p - q} = \frac{-2 - 3}{-2 - (-3)}$
 $= -5$ ✓ (1 mark)

d $pq^2 - p + q$
 $= -2 \times (-3)^2 - (-2) + (-3)$
 $= -2 \times 9 + 2 - 3$
 $= -19$ ✓ (1 mark)

WORKED SOLUTIONS



CHECK YOUR SOLUTIONS

5 a $s = ut + \frac{1}{2}at^2$
 When $u = 32$, $a = 9.8$ and $t = 7$,
 $s = 32 \times 7 + \frac{1}{2} \times 9.8 \times 7^2$
 $= 464.1$ ✓ (1 mark)

b $s = ut + \frac{1}{2}at^2$
 When $s = 1599$, $u = 58$ and $t = 13$,
 $1599 = 58 \times 13 + \frac{1}{2} \times a \times 13^2$
 $1599 = 754 + 84.5a$ ✓
 $84.5a = 845$
 $a = 10$ ✓ (2 marks)

6 a $V = \frac{4}{3}\pi r^3$
 When $r = 9$,
 $V = \frac{4}{3} \times \pi \times 9^3$
 $= 3053.62805 \dots$
 $= 3053.63$ (2 d.p.) ✓
 (1 mark)

b $V = \frac{4}{3}\pi r^3$
 When $V = 222665$,
 $222665 = \frac{4}{3}\pi r^3$
 $r^3 = 222665 \div \frac{4}{3}\pi$
 $r = \sqrt[3]{222665 \div \frac{4}{3}\pi}$ ✓
 $= 37.599994 \dots$
 $= 37.60$ (2 d.p.) ✓
 (2 marks)

7 a $F = \frac{9C}{5} + 32$
 When $C = 180$,
 $F = \frac{9 \times 180}{5} + 32$
 $= 356$
 The oven should be set at a temperature of 356°F . ✓
 (1 mark)

b $F = \frac{9C}{5} + 32$
 When $F = 68$,
 $68 = \frac{9C}{5} + 32$
 $36 = \frac{9C}{5}$
 $9C = 180$
 $C = 20$

The temperature was only 20°C , which is not hot, so Tina was not correct. ✓
 (1 mark)

8 a $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 $d = 10$ when $x_1 = -3$, $y_1 = 2$ and $x_2 = 5$,
 $10 = \sqrt{(5 - (-3))^2 + (y_2 - 2)^2}$
 $10 = \sqrt{64 + (y_2 - 2)^2}$ ✓
 $100 = 64 + (y_2 - 2)^2$
 $36 = (y_2 - 2)^2$
 So $y_2 - 2 = 6$ or $y_2 - 2 = -6$
 $y_2 = 8$ or $y_2 = -4$ ✓
 (2 marks)

b There are two points on the line $x = 5$ that are 10 units from the point $(-3, 2)$. The points are $(5, -4)$ and $(5, 8)$. ✓ (1 mark)
 (Total: 40 marks)

FURTHER EQUATIONS AND INEQUALITIES SKILLS CHECK

PAGE 34

- 1 a** $3 > 2$ True
b $-3 > -2$ False
c $6 < 15$ True
d $-6 < -15$ False
e $4 \geq -1$ True
f $-4 \geq 1$ False
g $-2 \leq 2$ True
h $2 \leq -2$ False

- 2 a** $-3 \geq 2$ False
b $5 < 2 \times 3$
 $5 < 6$ True
c $12 + 8 \neq 35 - 5 \times 3$
 $20 \neq 35 - 15$
 $20 \neq 20$ False
d $19 > 17 - 3 \times -2$
 $19 > 17 + 6$
 $19 > 23$ False
e $6^2 \leq 2^6$
 $36 \leq 64$ True
f $8 + 5 \neq 5 + 8$
 $13 \neq 13$ True

- 3 a** $5a - 4 < 2a$
 Substitute $a = -2$

LHS $= 5 \times -2 - 4$
 $= -14$
 RHS $= 2 \times -2$
 $= -4$
 $-14 < -4$ True

b $\frac{9e-1}{11} \geq 7$
 Substitute $e = 5$
 LHS $= \frac{9 \times 5 - 1}{11}$
 $= 4$
 $4 < 7$ (not $\geq$) False

c $3p + 1 \leq 3 - 3p$
 Substitute $p = -1$
 LHS $= 3 \times -1 + 1$
 $= -3 + 1$
 $= -2$
 RHS $= 3 - 3 \times -1$
 $= 3 - (-3)$
 $= 6$
 $-2 \leq 6$ True

d $\frac{x}{3} + 8 \geq -2$
 Substitute $x = -30$
 LHS $= \frac{-30}{3} + 8$
 $= -10 + 8$
 $= -2$
 $-2 \geq -2$ True

- 4 a** $-4p = 8$ [divide by -4]
 $p = -2$
b $4p < -8$
 $p < -2$
c $-4p > 8$
 $p < -2$ [dividing both sides by -4 , the inequality sign must be reversed]
d $3x + 17 = 74$
 $3x = 57$
 $x = 19$
e $2n - 1 > 15$
 $2n > 16$
 $n > 8$
f $5x \leq x + 8$
 $4x \leq 8$ [subtracting x from both sides]
 $x \leq 2$
g $\frac{a}{4} + 7 \geq -2$
 $\frac{a}{4} \geq -9$
 $a \geq -36$

WORKED SOLUTIONS



CHECK YOUR SOLUTIONS

$$\begin{aligned} \text{h } \frac{t-3}{2} &> -1 \\ t-3 &> -2 \\ t &> 1 \end{aligned}$$

$$\text{5 a } x+4 \leq -2 \\ x \leq -6$$

$$\text{b } a-5 > 5 \\ a > 10$$

$$\text{c } 4e < -2 \\ e < -\frac{1}{2} \quad \left[-\frac{2}{4}\right]$$

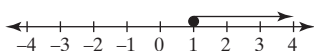
$$\text{d } \frac{n}{3} \geq 6 \\ n \geq 18 \quad [6 \times 3]$$

$$\text{6 a } 2x+5 \geq 11 \\ 2x \geq 11-5 \\ 2x \geq 6 \\ x \geq 3$$

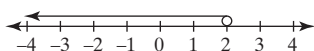
$$\text{b } 3a-1 < -2 \\ 3a < -2+1 \\ 3a < -1 \\ a < -\frac{1}{3}$$

$$\text{c } \frac{m+2}{3} \leq 4 \\ m+2 \leq 4 \times 3 \\ m+2 \leq 12 \\ m \leq 12-2 \\ m \leq 10$$

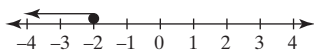
$$\text{7 a } x \geq 1$$



$$\text{b } x < 2$$



$$\text{c } x \leq -2$$



$$\begin{aligned} \text{8 } \text{If } x &= -1, \\ 5(x+1) - 2(3-2x) & \\ = 5(-1+1) - 2(3-2 \times -1) & \\ = -10 \quad (\neq 10) & \quad \text{False} \end{aligned}$$

$$\begin{aligned} \text{9 } \text{If } x &= -1, \\ 7x-3 &= 7 \times -1-3 \\ &= -10 \\ x-8 &= -1-8 \\ &= -9 \quad \text{False} \\ [\text{because } -10 &\neq -9] \end{aligned}$$

$$\begin{aligned} \text{10 a } A &= 36, b = 8 \\ A &= \frac{1}{2}bh \\ 36 &= \frac{1}{2} \times 8 \times h \\ &= 4h \\ h &= 36 \div 4 \\ &= 9 \end{aligned}$$

$$\begin{aligned} \text{b } A &= 84, h = 7 \\ A &= \frac{1}{2}bh \\ 84 &= \frac{1}{2} \times b \times 7 \\ &= 3.5b \\ b &= 84 \div 3.5 \\ &= 24 \end{aligned}$$

The base is 24 cm long.

FURTHER EQUATIONS AND INEQUALITIES

INTERMEDIATE TEST PAGE 35

$$\begin{aligned} \text{1 } 11 &< 2x-15 \quad [\text{add } 15] \\ 26 &< 2x \quad [\text{divide by } 2] \\ 13 &< x \\ x &> 13 \end{aligned}$$

None of the answers in A, B or C are greater than 13. [Instead of solving you could substitute each answer to see if it satisfies the inequality.]

$\therefore$ [D] ✓ (1 mark)

$$\begin{aligned} \text{2 } \frac{7m}{3} - 8 &\leq 9 - \frac{m}{2} \\ \text{Substitute } m &= -2, -12 \frac{2}{3} < 10 \quad (\text{True}) \end{aligned}$$

Substitute $m = 0$, $-8 \leq 9$ (True)

Substitute $m = 6$, $6 \leq 6$ (True)

Substitute $m = 9$, $13 \leq 4 \frac{1}{2}$ (False)

$$\begin{aligned} [\text{Or } \frac{7m}{3} - 8 &\leq 9 - \frac{m}{2} \\ (\text{multiply both sides by } 6) & \end{aligned}$$

$$14m - 48 \leq 54 - 3m$$

$$17m \leq 102$$

$$m \leq 6$$

The only value of m not less than or equal to 6 is $m = 9$.]

$\therefore$ [D] ✓ (1 mark)

$$\begin{aligned} \text{3 } -2x &> -6 \quad [\text{divide by } -2] \\ x &< 3 \\ [\text{The sign must be reversed} & \text{ because we divide by a negative number.}] \\ \therefore \text{ [B] } & \checkmark \quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{4 } 3p-17 &= 5p+19 \quad [\text{add } 17] \\ 3p &= 5p+36 \quad [\text{subtract } 5p] \\ -2p &= 36 \quad [\text{divide by } -2] \\ p &= -18 \\ \therefore \text{ [A] } & \checkmark \quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{5 } \text{Let the number be } x. \\ \text{Three times the number} &= 3 \times x \\ &= 3x \\ \text{Subtracting } 15 \text{ from } 3x &\text{ gives } 3x-15 \\ \text{Twice the number} &= 2 \times x = 2x \\ \text{8 more than twice the number} &= 2x+8 \\ 3x-15 &= 2x+8 \\ \therefore \text{ [A] } & \checkmark \quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{6 } \text{The number 8 more than } 2x-5 &= 2x-5+8 \\ &= 2x+3 \\ \therefore \text{ [A] } & \checkmark \quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{7 a } \text{Let } x &\text{ be the price of a bottle of soft drink.} \\ 7x + \$2.75 &= \$15.98 \quad \checkmark \\ \therefore \text{ [A] } & \checkmark \quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{b } 7x + \$2.75 &= \$15.98 \\ 7x &= \$13.23 \\ x &= \$1.89 \\ \text{One bottle of soft drink} &\text{ costs } \$1.89. \quad \checkmark \\ \text{The cost of 5 bottles} &= 5 \times \$1.89 \\ &= \$9.45 \quad \checkmark \quad (2 \text{ marks}) \end{aligned}$$

$$\begin{aligned} \text{8 } 5(2x+3) &\leq 20 \\ 10x+15 &\leq 20 \\ 10x &\leq 5 \\ x &\leq \frac{1}{2} \quad \checkmark \\ [\text{Or } 5(2x+3) &\leq 20 \\ 2x+3 &\leq 4 \\ 2x &\leq 1 \\ x &\leq \frac{1}{2}] \quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{9 } 9n+4 &> 2n-17 \\ 7n+4 &> -17 \quad \checkmark \\ 7n &> -21 \\ n &> -3 \quad \checkmark \quad (2 \text{ marks}) \end{aligned}$$

WORKED SOLUTIONS



CHECK YOUR SOLUTIONS

10 $3 - 2a > 7$
 $-2a > 4$ ✓
 $a < -2$ ✓

(3 marks)

11 $3m + 17 \leq 8m$
 $17 \leq 5m$ ✓
 $3.4 \leq m$
 $m \geq 3.4$ ✓
 [Or: $3m + 17 \leq 8m$
 $3m \leq 8m - 17$
 $-5m \leq -17$
 $m \geq 3.4$] (2 marks)

12 $2 - \frac{u}{7} \geq 3$
 $-\frac{u}{7} \geq 1$ ✓
 $\frac{u}{7} \leq -1$ ✓
 [Or multiply each term by 7 first.
 $14 - u \geq 21$
 $-u \geq 7$
 $u \leq -7$] (2 marks)

13 Let x be the number.
 $5(x - 15) = 3x + 7$ ✓
 $5x - 75 = 3x + 7$
 $2x - 75 = 7$ ✓
 $2x = 82$
 $x = 41$
 The number is 41. ✓ (3 marks)

14 Let the width be x cm.

 The length is $(x + 20)$ cm
 $P = 2(x + 20) + 2x$
 $= 2x + 40 + 2x$
 $= 4x + 40$ ✓
 Now $4x + 40 \leq 150$ ✓
 $4x \leq 110$
 $x \leq 27.5$
 If $x = 27.5$, $x + 20 = 27.5 + 20$
 $= 47.5$
 The largest rectangle is 27.5 cm wide and 47.5 cm long. ✓
 (3 marks)
 (Total: 25 marks)

FURTHER EQUATIONS AND INEQUALITIES

ADVANCED TEST

PAGE 36

1 a $x \geq 3$

✓ (1 mark)

b $x < -1$ or $x > 4$

✓ (1 mark)

c $-2 \leq x < 5$

✓ (1 mark)

2 a $5x + 1 \geq 9 - 3x$
 $8x \geq 8$
 $x \geq 1$ ✓

✓ (2 marks)

b $3(p + 5) - 1 < 11$
 $3p + 15 - 1 < 11$
 $3p < -3$
 $p < -1$ ✓

✓ (2 marks)

c $9 - 2n > 5$
 $-2n > -4$
 $n < 2$ ✓

✓ (2 marks)

3 a $3(2m - 5) \leq 2(m + 7)$
 $6m - 15 \leq 2m + 14$ ✓
 $4m \leq 29$
 $m \leq 7.25$ ✓ (2 marks)

b $\frac{5k}{3} - 2 < 4 - k$
 $5k - 6 < 12 - 3k$ ✓
 $8k < 18$
 $k < 2.25$ ✓ (2 marks)

c $\frac{6 - a}{2} > 4$
 $6 - a > 8$
 $-a > 2$ ✓
 $a < -2$ ✓ (2 marks)

d $7(4q - 1) - 3(2q + 5) \geq 0$
 $28q - 7 - 6q - 15 \geq 0$ ✓
 $22q \geq 22$
 $q \geq 1$ ✓
 (2 marks)

e $12 - \frac{3d}{4} \leq 18$
 $48 - 3d \leq 72$
 $-3d \leq 24$ ✓
 $d \geq -8$ ✓ (2 marks)

f $5(2y - 3) - 4(3y - 7) > 2(y + 5)$
 $10y - 15 - 12y + 28 > 2y + 10$ ✓
 $-2y + 13 > 2y + 10$
 $-4y > -3$
 $y < \frac{3}{4}$ ✓
 (2 marks)

4 Jo divided by x without considering whether $x = 0$ is a solution to the equation. ✓
 $x(x + 5) = x(x + 3)$
 $x^2 + 5x = x^2 + 3x$
 $5x = 3x$
 $2x = 0$
 $x = 0$ ✓ (2 marks)

5 a $y = mx + b$
 $b = y - mx$ ✓ (1 mark)

b $A = \frac{1}{2}h(a + b)$
 $2A = h(a + b)$
 $a + b = \frac{2A}{h}$ ✓
 $b = \frac{2A}{h} - a$ ✓ (2 marks)

c $c = \sqrt{a^2 + b^2}$
 $c^2 = a^2 + b^2$
 $b^2 = c^2 - a^2$ ✓
 $b = \pm \sqrt{c^2 - a^2}$ ✓ (2 marks)

6 a $V = \pi r^2 h$
 $r^2 = \frac{V}{\pi h}$
 $r = \pm \sqrt{\frac{V}{\pi h}}$ ✓ (1 mark)

b There are no restrictions on the variables before the formula is rearranged, although in practical situations, because it is the formula for the volume of a cylinder, V , r and h would all be greater than 0. ✓
 After it is rearranged, $h \neq 0$ and $\frac{V}{h} > 0$. ✓ (2 marks)

WORKED SOLUTIONS



CHECK YOUR SOLUTIONS

c As $r > 0$, $r = \sqrt{\frac{V}{\pi h}}$
 When $V = 179$ and $h = 8.7$,

$$r = \sqrt{\frac{179}{\pi \times 8.7}}$$

$$= 2.559127\dots$$

$$= 2.6 \text{ (1 d.p.)} \quad \checkmark \text{ (1 mark)}$$

7 a $A = \frac{1}{2}bh$

$$27.3 = \frac{1}{2} \times 19.5 \times h$$

$$9.75h = 27.3$$

$$h = \frac{27.3}{9.75}$$

$$= 2.8$$

 The perpendicular height is 2.8 m. $\checkmark$ (1 mark)

b Let x m be the length of the hypotenuse.

$$x^2 = 19.5^2 + 2.8^2$$

$$= 388.09$$

$$x = \sqrt{388.09} \quad (x > 0)$$

$$= 19.7$$

 The hypotenuse is 19.7 m long. $\checkmark$ (1 mark)

c The length of any side of a triangle cannot be negative. It makes no sense for any length to be negative. $\checkmark$ (1 mark)

8 a When $x = -23$,

$$\sqrt{3 - 2x} = \sqrt{3 - 2 \times (-23)}$$

$$= 7 \quad \checkmark \text{ (1 mark)}$$

b $3 - 2x \geq 0$

$$-2x \geq -3$$

$$x \leq 1.5 \quad \checkmark \text{ (1 mark)}$$

c $\sqrt{3 - 2x} = 5$

$$3 - 2x = 25$$

$$-2x = 22$$

$$x = -11 \quad \checkmark \text{ (1 mark)}$$

d i $\sqrt{3 - 2x} > 5$ when $x < -11$ $\checkmark$ (1 mark)
 [From part c we can see that the solution is either $x < -11$ or $x > -11$; from part a we can see that values of x less than -11 give values of the square root greater than 5.]

ii $\sqrt{3 - 2x} < 5$ when $-11 < x \leq 1.5$ $\checkmark$ (1 mark)

[Values of x greater than -11 will give values of the square root less than 5 but from part b we know that x cannot be greater than 1.5.]
 (Total: 40 marks)

QUADRATIC EQUATIONS

SKILLS CHECK

PAGE 38

1 a If $x = 3$, $x^2 = 3^2$

$$= 9$$

b If $x = -3$, $x^2 = (-3)^2$

$$= 9$$

c If $x = 7$, $x^2 = 7^2$

$$= 49$$

d If $x = -7$, $x^2 = (-7)^2$

$$= 49$$

e If $x = -5$, $x^2 = (-5)^2$

$$= 25$$

f If $x = 5$, $x^2 = 5^2$

$$= 25$$

g If $x = 1$, $x^2 = 1^2$

$$= 1$$

h If $x = -1$, $x^2 = (-1)^2$

$$= 1$$

2 a $(-6)^2 = 36$
 The other number is -6 .

b $10^2 = 100$ and $(-10)^2 = 100$
 The numbers are 10 and -10 .

3 a $x^2 = 9$
 $x = 3$ or $x = -3$

b $x^2 = 49$
 $x = 7$ or $x = -7$

c $x^2 = 25$
 $x = 5$ or $x = -5$

d $x^2 = 1$
 $x = 1$ or $x = -1$

4 a If $p = 2$, $2p^2 = 2 \times 2^2$

$$= 2 \times 4$$

$$= 8$$

b If $p = -2$, $2p^2 = 2 \times (-2)^2$

$$= 2 \times 4$$

$$= 8$$

c If $p = 6$, $2p^2 = 2 \times 6^2$

$$= 2 \times 36$$

$$= 72$$

d If $p = -6$, $2p^2 = 2 \times (-6)^2$

$$= 2 \times 36$$

$$= 72$$

5 a $2p^2 = 8$

$$p^2 = 4$$

$$p = 2 \text{ or } p = -2$$

b $2p^2 = 72$

$$p^2 = 36$$

$$p = 6 \text{ or } p = -6$$

6 a $x^2 = 16$
 $x = 4$ or $x = -4$

b $3e^2 = 243$

$$e^2 = 81$$

$$e = 9 \text{ or } e = -9$$

c $4k^2 = 1$

$$k^2 = \frac{1}{4}$$

$$k = \frac{1}{2} \text{ or } k = -\frac{1}{2}$$

7 a $x^2 = 60$

$$x = \pm\sqrt{60}$$

$$= \pm 7.74596669\dots$$

$$= \pm 7.75 \quad (2 \text{ d.p.})$$

b $x^2 = 12.5$

$$x = \pm\sqrt{12.5}$$

$$= \pm 3.535533906\dots$$

$$= \pm 3.54 \quad (2 \text{ d.p.})$$

c $7x^2 = 45$

$$x^2 = \frac{45}{7} = 6.4285714\dots$$

$$x = \pm\sqrt{6.4285714\dots}$$

$$= \pm 2.535462764\dots$$

$$= \pm 2.54 \quad (2 \text{ d.p.})$$

8 a $5x = 0$ [divide both sides by 5]

$$x = 0$$

b $3a = 0$

$$a = 0$$

c $12t = 0$

$$t = 0$$

d $-37m = 0$
 [divide both sides by -37]

$$m = 0$$

9 a $3(x - 7) = 0$

$$x - 7 = 0$$

$$x = 7$$

b $4(2x + 5) = 0$

$$2x + 5 = 0$$

$$2x = -5$$

$$x = -2.5$$

c $6(3x - 2) = 0$

$$3x - 2 = 0$$

$$3x = 2$$

$$x = \frac{2}{3}$$

WORKED SOLUTIONS



CHECK YOUR SOLUTIONS

- 10 a** $x(x+4) = 0$
 $x = 0$ or $x + 4 = 0$
 $x = 0$ or $x = -4$
- b** $(x-7)(x+1) = 0$
 $x-7 = 0$ or $x+1 = 0$
 $x = 7$ or $x = -1$
- c** $(2a+5)(3a-2) = 0$
 $2a+5 = 0$ or $3a-2 = 0$
 $2a = -5$ or $3a = 2$
 $a = -\frac{5}{2}$ or $a = \frac{2}{3}$
- 11 a** $x^2 - 9x = x(x-9)$
- b** $x^2 + 8x + 15 = (x+5)(x+3)$
 [What 2 numbers add to 8 and multiply to 15?]
- c** $t^2 + 3t - 10 = (t+5)(t-2)$
 [What 2 numbers add to 3 and multiply to -10?]
- d** $m^2 - 7m + 12 = (m-3)(m-4)$
 [Or $(m-4)(m-3)$]
- e** $n^2 - 4n - 60 = (n-10)(n+6)$
 [Or $(n+6)(n-10)$]
- f** $a^2 - 2a + 1 = (a-1)(a-1)$
 $= (a-1)^2$
- 12 a** When $x = 3$,
 $x^2 + 7x + 12 = 3^2 + 7 \times 3 + 12 = 42 \neq 0$ False
- b** When $y = -3$,
 $y^2 - 4y - 21 = (-3)^2 - 4 \times -3 - 21 = 0$ True
- 13 a** $n^2 - 3n = 0$
 $n(n-3) = 0$
 $n = 0$ or $n-3 = 0$
 $n = 0$ or $n = 3$
- b** $x^2 + 6x + 8 = 0$
 $(x+2)(x+4) = 0$
 $x+2 = 0$ or $x+4 = 0$
 $x = -2$ or $x = -4$
- c** $a^2 - 4a - 45 = 0$
 $(a-9)(a+5) = 0$
 $a-9 = 0$ or $a+5 = 0$
 $a = 9$ or $a = -5$

QUADRATIC EQUATIONS

INTERMEDIATE TEST PAGE 39

- 1** $4a^2 = 16$
 $a^2 = 4$
 $a = \pm 2$
 Both $a = 2$ and $a = -2$ are correct.
 $\therefore$ [D] ✓ (1 mark)
- 2** $p^2 + 8p - 9 = 0$
 $(p+9)(p-1) = 0$
 $p+9 = 0$ or $p-1 = 0$
 $p = -9$ or $p = 1$
 $p = -9$ is a solution.
 [Or substitute each value into the equation to see which solution satisfies it.]
 $\therefore$ [D] ✓ (1 mark)
- 3** $x^2 - 11x + 28 = 0$
 $(x-4)(x-7) = 0$
 $x-4 = 0$ or $x-7 = 0$
 $x = 4$ or $x = 7$
 $\therefore$ [A] ✓ (1 mark)
- 4** $x = 3$ or $x = -2$
 $x-3 = 0$ or $x+2 = 0$
 $(x-3)(x+2) = 0$
 $x^2 + 2x - 3x - 6 = 0$
 $x^2 - x - 6 = 0$
 $\therefore$ [D] ✓ (1 mark)
- 5** $(x-5)(x-2) = 28$
 $x^2 - 2x - 5x + 10 = 28$
 $x^2 - 7x - 18 = 0$
 $(x+2)(x-9) = 0$
 $x+2 = 0$ or $x-9 = 0$
 $x = -2$ or $x = 9$
 $\therefore$ [C] ✓ (1 mark)
 [Or substitute to see which solutions work.]
- 6** $100x^2 = 1$
 $x^2 = 0.01$
 $x = \pm\sqrt{0.01}$
 $x = \pm 0.1$ ✓✓
 [Or $\pm\frac{1}{10}$] (2 marks)
- 7** $4x^2 - 100 = 0$
 $4x^2 = 100$
 $x^2 = 25$
 $x = \pm 5$ ✓✓ (2 marks)

- 8 a** $x^2 + 11x = 0$
 $x(x+11) = 0$
 $x = 0$ or $x+11 = 0$
 $x = 0$ or $x = -11$
 ✓✓ (2 marks)
- b** $x^2 + 2x - 80 = 0$
 $(x+10)(x-8) = 0$
 $x+10 = 0$ or $x-8 = 0$
 $x = -10$ or $x = 8$
 ✓✓ (2 marks)
- c** $x^2 - 18x + 81 = 0$
 $(x-9)^2 = 0$ ✓
 $x-9 = 0$ ✓
 $x = 9$ ✓
 (2 marks)
- d** $x^2 + 14x + 48 = 0$
 $(x+6)(x+8) = 0$
 $x+6 = 0$ or $x+8 = 0$
 $x = -6$ or $x = -8$
 ✓✓ (2 marks)
- 9** $(2x+3)^2 = 25$
 $2x+3 = 5$ or $2x+3 = -5$
 $2x = 2$ or $2x = -8$
 $x = 1$ or $x = -4$
 ✓✓ (2 marks)
- 10 a** Because the two factors multiply to 35 it doesn't mean that one must be 35. ✓
 The rule only works if the product is zero. If one of the factors was 35 the other would have to be 1 and that is not possible. ✓ (2 marks)
- b** $(x+3)(x+5) = 35$
 $x^2 + 5x + 3x + 15 = 35$
 $x^2 + 8x - 20 = 0$ ✓
 $(x+10)(x-2) = 0$
 $x+10 = 0$ or $x-2 = 0$
 $x = -10$ or $x = 2$
 ✓✓ (3 marks)
- 11 a** Let the width be x m.
 The length = $(x+12)$ m ✓
 $A = lb$
 $= x(x+12)$ ✓
 But the area is 45 m^2
 $x(x+12) = 45$
 $x^2 + 12x - 45 = 0$ ✓
 The width, x m, can be found by solving that equation. (3 marks)

WORKED SOLUTIONS



CHECK YOUR SOLUTIONS

b $x^2 + 12x - 45 = 0$
 $(x-3)(x+15) = 0$
 $x-3 = 0$ or $x+15 = 0$
 $x = 3$ or $x = -15$ ✓
 But $x > 0$, [because it is the width of the rectangle]
 $\therefore x = 3$ ✓
 $l = x + 12$
 $= 3 + 12$
 $= 15$
 The length of the rectangle is 15 m and the width is 3 m. ✓ (3 marks)
(Total: 30 marks)

QUADRATIC EQUATIONS

ADVANCED TEST

PAGE 40

1 a $a^2 = \frac{1}{16}$
 $a = \pm \frac{1}{4}$ ✓ (1 mark)

b $4t^2 = 25$
 $t^2 = 6.25$
 $t = \pm 2.5$ ✓ (1 mark)

c $12d^2 - 27 = 0$
 $12d^2 = 27$
 $d^2 = 2.25$
 $d = \pm 1.5$ ✓ (1 mark)

2 If $x^2 + 1 = 0$ then $x^2 = -1$
 The square of a real number is always positive so there is no real solution. ✓ (1 mark)

3 a $x^2 + 9x + 14 = 0$
 $(x+7)(x+2) = 0$
 $x+7 = 0$ or $x+2 = 0$
 $x = -7$ or $x = -2$ ✓ (1 mark)

b $a^2 - 2a - 48 = 0$
 $(a-8)(a+6) = 0$
 $a-8 = 0$ or $a+6 = 0$
 $a = 8$ or $a = -6$ ✓ (1 mark)

c $54 - 3x - x^2 = 0$
 $(9+x)(6-x) = 0$
 $9+x = 0$ or $6-x = 0$
 $x = -9$ or $x = 6$ ✓ (1 mark)

4 a $x^2 = 5x$
 $x^2 - 5x = 0$
 $x(x-5) = 0$ ✓
 $x = 0$ or $x = 5$ ✓ (2 marks)

b $n^2 = 12n + 64$
 $n^2 - 12n - 64 = 0$
 $(n-16)(n+4) = 0$ ✓
 $n = 16$ or $n = -4$ ✓ (2 marks)

c $s^2 + 5s = 24$
 $s^2 + 5s - 24 = 0$
 $(s-3)(s+8) = 0$ ✓
 $s = 3$ or $s = -8$ ✓ (2 marks)

5 a $x^2 = 7$
 $x = \pm\sqrt{7}$ ✓
 $= \pm 2.64575 \dots$
 $= \pm 2.65$ (2 d.p.) ✓ (2 marks)

b $3y^2 = 11$
 $y^2 = \frac{11}{3}$
 $y = \pm \sqrt{\frac{11}{3}}$ ✓
 $= \pm 1.914854 \dots$
 $= \pm 1.91$ (2 d.p.) ✓ (2 marks)

c $5m^2 - 13 = 0$
 $5m^2 = 13$
 $m^2 = 2.6$
 $m = \pm\sqrt{2.6}$ ✓
 $= \pm 1.612451 \dots$
 $= \pm 1.61$ (2 d.p.) ✓ (2 marks)

6 a $2x^2 - 28x + 66 = 0$
 $x^2 - 14x + 33 = 0$
 $(x-3)(x-11) = 0$ ✓
 $x = 3$ or $x = 11$ ✓ (2 marks)

b $3m^2 + 30m - 72 = 0$
 $m^2 + 10m - 24 = 0$
 $(m+12)(m-2) = 0$ ✓
 $m = -12$ or $m = 2$ ✓ (2 marks)

c $5y^2 - 5y - 10 = 0$
 $y^2 - y - 2 = 0$
 $(y-2)(y+1) = 0$ ✓
 $y = 2$ or $y = -1$ ✓ (2 marks)

7 a $y^3 + 7y^2 = 0$
 $y^2(y+7) = 0$
 $y^2 = 0$ or $y+7 = 0$ ✓
 $y = 0$ or $y = -7$ ✓ (2 marks)

b $a^3 - 9a = 0$
 $a(a^2 - 9) = 0$
 $a = 0$ or $a^2 = 9$ ✓
 $a = 0$ or $a = \pm 3$ ✓ (2 marks)

c $x^4 - 5x^2 + 4 = 0$
 $(x^2-4)(x^2-1) = 0$
 $x^2 = 4$ or $x^2 = 1$ ✓
 $x = \pm 2$ or $x = \pm 1$ ✓ (2 marks)

8 a $(x-5)(x-2) = -2$
 $x^2 - 7x + 10 = -2$
 $x^2 - 7x + 12 = 0$ ✓
 $(x-3)(x-4) = 0$ ✓
 $x = 3$ or $x = 4$ ✓ (2 marks)

b $(x+6)(x-7) = x+21$
 $x^2 - x - 42 = x+21$
 $x^2 - 2x - 63 = 0$ ✓
 $(x-9)(x+7) = 0$
 $x = 9$ or $x = -7$ ✓ (2 marks)

c $2x^2 = (x-5)(x-12)$
 $2x^2 = x^2 - 17x + 60$
 $x^2 + 17x - 60 = 0$ ✓
 $(x+20)(x-3) = 0$
 $x = -20$ or $x = 3$ ✓ (2 marks)

9 $(x+4)(x+6) + (x+8)(x-3) = (x-12)(x+2)$
 $x^2 + 10x + 24 + x^2 + 5x - 24 = x^2 - 10x - 24$ ✓
 $x^2 + 25x + 24 = 0$ ✓
 $(x+24)(x+1) = 0$
 $x = -24$ or $x = -1$ ✓ (3 marks)
(Total: 40 marks)

SIMULTANEOUS EQUATIONS

SKILLS CHECK

PAGE 42

1 a $3a + 11 = -1$
 $3a = -12$ [subtracting 11]
 $a = -4$ [dividing by 3]

b $4x + 3(6x-5) - 2 = 16$
 $4x + 18x - 15 - 2 = 16$
 $22x - 17 = 16$
 $22x = 33$
 $x = 1.5$

2 a $p + q = 5$
 When $q = -2$, $p - 2 = 5$
 $p = 7$

b $3p - q = 17$
 When $q = -2$, $3p + 2 = 17$
 $3p = 15$
 $p = 5$

WORKED SOLUTIONS



CHECK YOUR SOLUTIONS

c $4p + 3q - 10 = 0$

When $q = -2$,

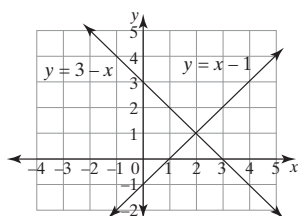
$$4p + 3 \times -2 - 10 = 0$$

$$4p - 16 = 0$$

$$4p = 16$$

$$p = 4$$

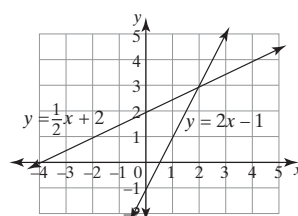
3 a



The lines intersect when $x = 2$ and $y = 1$.

[or at the point (2, 1)]

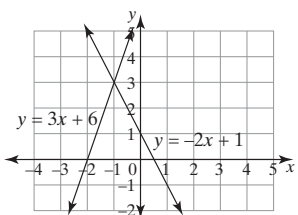
b



The lines intersect when $x = 2$ and $y = 3$.

[or at the point (2, 3)]

c



The lines intersect when $x = -1$ and $y = 3$.

[or at the point (-1, 3)]

4 $y = 2x - 5$

x	1	2	3	4	5	6	7
y	-3	-1	1	3	5	7	9

[continuing the pattern]

$$y = x + 1$$

x	1	2	3	4	5	6	7
y	2	3	4	5	6	7	8

The values of x and y that satisfy both equations are $x = 6$ and $y = 7$.

5 a $y = 3x - 8$

x	0	1	2	3	4	5	6
y	-8	-5	-2	1	4	7	10

b $x = y$ when $x = 4$ and $y = 4$

c $2x + y = 7$ when $x = 3$ and $y = 1$

6 a $7x - y = 10$
 $-y = -7x + 10$
 $y = 7x - 10$

b $5x + y - 2 = 0$
 $y = -5x + 2$

c $6x + 2y - 1 = 0$
 $2y = -6x + 1$
 $y = -3x + \frac{1}{2}$

7 a $2a + b = 5$ [i]
 $3a - b = 5$ [ii]
[i] + [ii] $5a = 10$

b $8x + 5y - 2 = 0$ [i]
 $3x - 5y + 13 = 0$ [ii]
[i] + [ii] $11x + 11 = 0$

c $7p - 3q = 15$ [i]
 $2p + 3q = 18$ [ii]
[i] + [ii] $9p = 33$

8 a $6x + 7y = 9$ [i]
 $2x + 7y = 13$ [ii]
[i] - [ii] $4x = -4$

b $11a - 3b = 8$ [i]
 $5a - 3b = 2$ [ii]
[i] - [ii] $6a = 6$

c $5a - 3b - 1 = 0$ [i]
 $5a + 6b + 10 = 0$ [ii]
[i] - [ii] $-9b - 11 = 0$

9 a Adding equation [i] to equation [ii] will eliminate b [giving $10a = 40$].

b Adding or subtracting. Adding equation [i] to equation [ii] will eliminate y [giving $16x = 34$]. Subtracting equation [ii] from equation [i] will eliminate x [giving $4y = 4$].

c Subtracting one equation from the other will eliminate q . [Subtracting [ii] from [i] will give $3p + 22 = 0$.]

10 a The number of blue pens plus the number of red pens equals 15.
 $x + y = 15$

b The number of blue pens is five more than the number of red pens.
 $x = y + 5$

c The number of blue pens is equal to two times the number of red pens.
 $x = 2y$

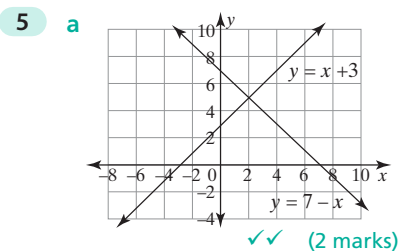
SIMULTANEOUS EQUATIONS INTERMEDIATE TEST PAGE 43

1 The point of intersection is where both $x = 3$ and $y = 4$.
i.e. at the point (3, 4)
 $\therefore$ [A] ✓ (1 mark)

2 $2x + y = 5$ [i]
 $x - 3y = 13$ [ii]
If $x = 4$ and $y = 3$
 $2x + y = 8 + 3$
 $= 11$
 $= 13$ ($\neq 5$)
If $x = 4$ and $y = -3$
 $2x + y = 8 - 3$
 $= 5$
 $x - 3y = 4 - 3 \times -3$
 $= 13$
[Also check that C and D do not work.]
 $\therefore$ [B] ✓ (1 mark)

3 $y = 4x + 7$ and $y = -2x - 5$
 $4x + 7 = -2x - 5$
 $6x = -12$
 $x = -2$
Substitute: $y = 4 \times -2 + 7$
 $= -1$
 $\therefore$ [A] ✓ (1 mark)

4 $y = x - 6$ and $y = -12x - 6$ are the only two equations with the same y -intercept.
So they have a simultaneous solution when $x = 0$.
 $\therefore$ [C] ✓ (1 mark)



b From the graph, $x = 2$ ✓
and $y = 5$ ✓
[Or the point (2, 5)]
(2 marks)

WORKED SOLUTIONS



CHECK YOUR SOLUTIONS

6 $7x - 2y = 5$ [i]
 $y = 3x$ [ii]
 Substitute [ii] into [i]
 $7x - 2(3x) = 5$
 $7x - 6x = 5$
 $x = 5$ ✓
 Substitute into [ii]
 $y = 3 \times 5$
 $= 15$ ✓
 $\therefore x = 5$ and $y = 15$
 [Check in (i),
 $7 \times 5 - 2 \times 15 = 5$] (2 marks)

7 $7a + 2b = 16$ [i]
 $3a - 2b = 24$ [ii]
 [i] + [ii] $10a = 40$
 $a = 4$ ✓
 Substitute in [i] [or into [ii]]
 $28 + 2b = 16$
 $2b = -12$
 $b = -6$ ✓
 $\therefore a = 4$ and $b = -6$
 [Check in [ii], $12 - (-12) = 24$] (2 marks)

8 a $2x + y - 5 = 0$
 $y = -2x + 5$ ✓
 (1 mark)

b $2x + y - 5 = 0$ [i]
 $3x - 4y + 42 = 0$ [ii]
 From a $y = -2x + 5$ [iii]
 Substitute into [ii]
 $3x - 4(-2x + 5) + 42 = 0$
 $3x + 8x - 20 + 42 = 0$
 $11x + 22 = 0$
 $11x = -22$
 $x = -2$ ✓
 Substitute into [iii]
 $y = -2 \times -2 + 5$
 $= 9$ ✓
 $\therefore x = -2$ and $y = 9$ (2 marks)

9 $9m + 4p = 41$ [i]
 $5m + 2p = 23$ [ii]
 [ii] $\times 2$ $10m + 4p = 46$ [iii]
 [iii] - [i] $m = 5$ ✓
 Substitute in [i]
 $9 \times 5 + 4p = 41$
 $45 + 4p = 41$
 $4p = -4$
 $p = -1$ ✓
 $\therefore m = 5$ and $p = -1$ (2 marks)

10 $2x - y + 8 = 0$ [i]
 $3x - y - 2 = 0$ [ii]

[ii] - [i] $x - 10 = 0$
 $x = 10$ ✓
 Substitute in [i]
 $2 \times 10 - y + 8 = 0$
 $28 - y = 0$
 $y = 28$ ✓
 $\therefore x = 10$ and $y = 28$
 [Or from [i] $y = 2x + 8$
 Substitute into [ii]
 $3x - (2x + 8) - 2 = 0$
 $x - 10 = 0$] (2 marks)

11 $5a + 3b = 8$ [i]
 $3a - 4b = 28$ [ii]
 [i] $\times 4$ $20a + 12b = 32$ [iii]
 [ii] $\times 3$ $9a - 12b = 84$ [iv] ✓
 [iii] + [iv] $29a = 116$
 $a = 4$ ✓
 Substitute into [i]
 $5 \times 4 + 3b = 8$
 $20 + 3b = 8$
 $3b = -12$
 $b = -4$ ✓ (3 marks)

12 Let x be the number of cans of dog food and y be the number of cans of cat food.
 $x + y = 24$ [i]
 $140x + 90y = 2910$ [ii] ✓
 From [i] $y = 24 - x$ [iii]
 Substitute into [ii]
 $140x + 90(24 - x) = 2910$
 $140x + 2160 - 90x = 2910$
 $50x = 750$
 $x = 15$ ✓
 Substitute into [iii]
 $y = 24 - 15$
 $= 9$ ✓
 There were 15 cans of dog food and 9 cans of cat food. ✓
 (4 marks)
 (Total: 26 marks)

SIMULTANEOUS EQUATIONS

ADVANCED TEST

PAGE 44

1 a $y = 5x - 6$ [i]
 $y = 3x + 2$ [ii]
 $5x - 6 = 3x + 2$
 $2x = 8$
 $x = 4$ ✓
 Substitute into [i]:
 $y = 5 \times 4 - 6$
 $= 14$
 $\therefore x = 4$ and $y = 14$ ✓
 (2 marks)

b $y = 7 - 2x$ [i]
 $x + 3y = 11$ [ii]
 Substitute [i] into [ii]
 $x + 3(7 - 2x) = 11$
 $x + 21 - 6x = 11$
 $-5x = -10$
 $x = 2$ ✓
 Substitute into [i]
 $y = 7 - 2 \times 2$
 $= 3$
 $\therefore x = 2$ and $y = 3$ ✓
 (2 marks)

c $3p - 5q = 23$ [i]
 $4p + q = 23$ [ii]
 From [ii] $q = 23 - 4p$
 Substitute into [i]
 $3p - 5(23 - 4p) = 23$
 $3p - 115 + 20p = 23$
 $23p = 138$
 $p = 6$ ✓
 Substitute into [ii]
 $q = 23 - 4 \times 6$
 $= -1$
 $\therefore p = 6$ and $q = -1$ ✓
 (2 marks)

2 a $a + b = 4$ [i]
 $a - b = -10$ [ii]
 [i] + [ii] $2a = -6$
 $a = -3$ ✓
 Substitute into [i]
 $-3 + b = 4$
 $b = 7$
 $\therefore a = -3$ and $b = 7$ ✓
 (2 marks)

b $8x - 3y = 30$ [i]
 $5x - 3y = 3$ [ii]
 [i] - [ii] $3x = 27$
 $x = 9$ ✓
 Substitute into [i]:
 $8 \times 9 - 3y = 30$
 $3y = 42$
 $y = 14$
 $\therefore x = 9$ and $y = 14$ ✓
 (2 marks)

c $9m + 5n - 3 = 0$ [i]
 $9m - 7n + 69 = 0$ [ii]
 [i] - [ii] $12n - 72 = 0$
 $12n = 72$
 $n = 6$ ✓
 Substitute into [i]
 $9m + 5 \times 6 - 3 = 0$
 $9m = -27$
 $m = -3$
 $\therefore m = -3$ and $n = 6$ ✓
 (2 marks)

WORKED SOLUTIONS



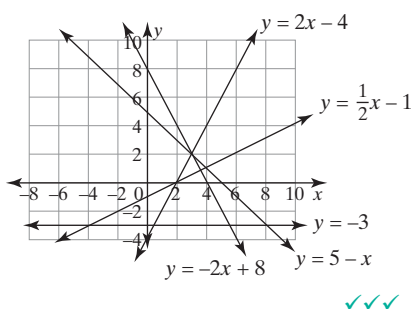
CHECK YOUR SOLUTIONS

d $4p + 3q = 27$ [i]
 $3p + q = 14$ [ii]
 [ii] $\times 3$: $9p + 3q = 42$ [iii]
 [iii] - [i] $5p = 15$
 $p = 3$ ✓
 Substitute into [ii]
 $3 \times 3 + q = 14$
 $q = 5$
 $\therefore p = 3$ and $q = 5$ ✓
 (2 marks)

e $9a - 4b = -1$ [i]
 $5a + 2b = -9$ [ii]
 [ii] $\times 2$: $10a + 4b = -18$ [iii]
 [i] + [iii] $19a = -19$
 $a = -1$ ✓
 Substitute into [i]:
 $9 \times -1 - 4b = -1$
 $4b = -8$
 $b = -2$
 $\therefore a = -1$ and $b = -2$ ✓
 (2 marks)

f $6k - 5m = 19$ [i]
 $7k - 3m = 42$ [ii]
 [i] $\times 3$: $18k - 15m = 57$ [iii]
 [ii] $\times 5$: $35k - 15m = 210$ [iv]
 [iv] - [iii] $17k = 153$
 $k = 9$ ✓
 Substitute into [i]
 $6 \times 9 - 5m = 19$
 $5m = 35$
 $m = 7$
 $\therefore k = 9$ and $m = 7$ ✓
 (2 marks)

3 a



- b i** (2, 0) ✓ (1 mark)
ii (4, 1) ✓ (1 mark)
iii (-4, -3) ✓ (1 mark)

c It is the point where $y = -2x + 8$, $y = 5 - x$ and $y = 2x - 4$ all intersect. ✓
 (1 mark)

4 a $x = x^2$
 $x - x^2 = 0$
 $x(1 - x) = 0$
 $x = 0$ or $x = 1$ ✓ (1 mark)

b When $x = 0$, $y = 0$ and when $x = 1$, $y = 1$. ✓ (1 mark)

c Two places; at (0, 0) and (1, 1). ✓ (1 mark)

d Two places. ✓ (1 mark)

e $x^2 = 4$
 $x = \pm 2$
 When $x = -2$, $y = 4$ and when $x = 2$, $y = 4$. ✓ (1 mark)

5 a $y = x^2 + 5x$ [i]
 $y = x + 12$ [ii]
 $x^2 + 5x = x + 12$
 $x^2 + 4x - 12 = 0$ ✓
 $(x + 6)(x - 2) = 0$
 $x = -6$ or $x = 2$
 Substitute into [ii]
 When $x = -6$, $y = 6$ ✓
 When $x = 2$, $y = 14$ ✓
 (3 marks)

b $y = x^2 + x - 10$ [i]
 $y = 2x - 4$ [ii]
 $x^2 + x - 10 = 2x - 4$
 $x^2 - x - 6 = 0$ ✓
 $(x - 3)(x + 2) = 0$
 $x = 3$ or $x = -2$
 Substitute into [ii]
 When $x = 3$, $y = 2 \times 3 - 4 = 2$ ✓
 When $x = -2$, $y = 2 \times -2 - 4 = -8$ ✓
 (3 marks)

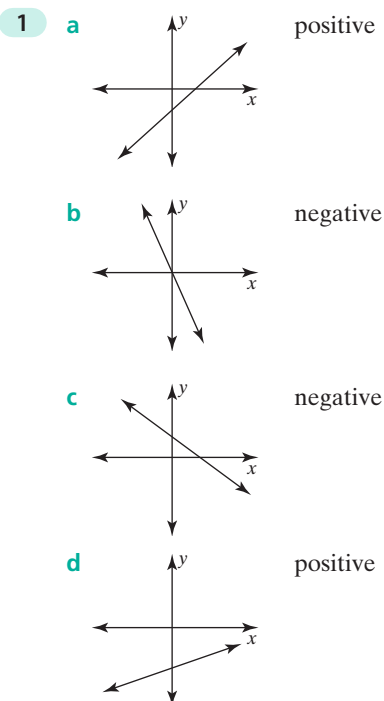
6 $y = x^2 + 8x + 3$ [i]
 $y = x^2 - 2x - 7$ [ii]
 $x^2 + 8x + 3 = x^2 - 2x - 7$
 $10x = -10$
 $x = -1$ ✓
 Substitute into [i]
 $y = (-1)^2 + 8 \times -1 + 3 = -4$
 $\therefore x = -1$ and $y = -4$ ✓
 (2 marks)

7 a You can immediately see the point of intersection once the graph is drawn, and easily see how many points of intersection there are. ✓ (1 mark)

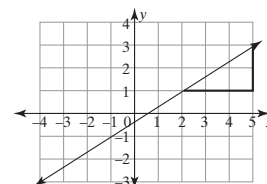
b Sketching a graph can be time-consuming and not always accurate. Solving equations algebraically will give the exact values. ✓
 (1 mark)
 (Total: 40 marks)

PARALLEL AND PERPENDICULAR LINES

PAGE 46



2 a $m = \frac{\text{rise}}{\text{run}}$
 $= \frac{2}{3}$

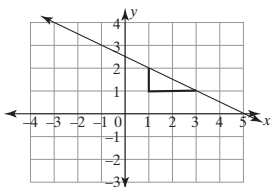


b $m = \frac{\text{rise}}{\text{run}}$
 $= \frac{-1}{2}$
 $= -\frac{1}{2}$

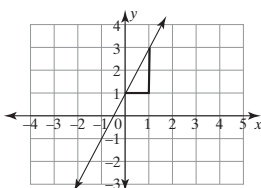
WORKED SOLUTIONS



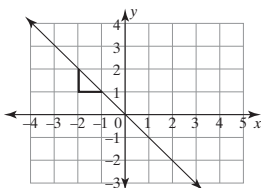
CHECK YOUR SOLUTIONS



$$\begin{aligned} c \quad m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{1}{1} \\ &= 1 \end{aligned}$$



$$\begin{aligned} d \quad m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{-1}{1} \\ &= -1 \end{aligned}$$



3 (1, 2), (5, 0)

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{0 - 2}{5 - 1} \\ &= \frac{-2}{4} \\ &= -\frac{1}{2} \end{aligned}$$

[Or plot the points and find the rise and run.]

- 4
- a $y = 2x + 3$
 $m = 2, b = 3$
 - b $y = 4x - 1$
 $m = 4, b = -1$
 - c $y = -6x$ [$y = -6x + 0$]
 $m = -6, b = 0$
 - d $y = x - 5$ [$y = 1x - 5$]
 $m = 1, b = -5$
 - e $y = \frac{1}{3}x - 2$
 $m = \frac{1}{3}, b = -2$

f $y = \frac{x}{4}$ [$y = \frac{1}{4}x + 0$]

$$m = \frac{1}{4}, b = 0$$

g $y = \frac{-2x}{3} + \frac{1}{2}$

$$m = \frac{-2}{3}, b = \frac{1}{2}$$

h $y = 5 - 2x$ [$y = -2x + 5$]

$$m = -2, b = 5$$

5 a $m = 3, b = 2$

$$y = 3x + 2$$

b $m = -2, b = 0$

$$y = -2x$$

c $m = \frac{1}{2}, b = -\frac{1}{3}$

$$y = \frac{1}{2}x - \frac{1}{3}$$

- 6 a If the gradients of two lines are equal then the lines are **parallel**.

- b If two lines are parallel then their gradients are **equal**.

- 7 a If $m_1 = m_2$ the two lines are **parallel**.

- b If $m_1 m_2 = -1$ the two lines are **perpendicular**.

8 a $-\frac{1}{2}$

[Parallel lines have the same gradient.]

- b 2
[The negative reciprocal of $-\frac{1}{2}$.]

- 9 a Both lines have gradient 3 so they are parallel. True.

- b Both lines have gradient $\frac{1}{2}$ so they are parallel. True.

- c Both lines have gradient -1 so they are parallel. True.

PARALLEL AND PERPENDICULAR LINES INTERMEDIATE TEST PAGE 47

- 1 $y = 3x - 4$ has gradient 3. Parallel lines have gradient 3. The line parallel to $y = 3x - 4$ is $y = 3x + 7$.
 $\therefore$ [D] ✓ (1 mark)

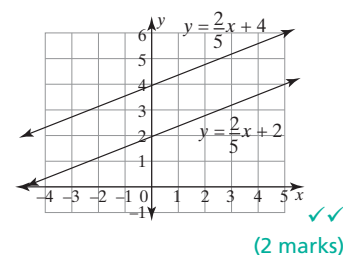
- 2 $y = 2x - 5$
Gradient = 2 [coefficient of x]
Any perpendicular line has gradient $-\frac{1}{2}$.
[the negative reciprocal of 2]
 $\therefore$ [D] ✓ (1 mark)

- 3 $y = 2 - x$
 $y = -x + 2$
Gradient = -1
Any parallel line has $m = -1$. Any line that passes through the origin has y -intercept 0. Equation is $y = -x$.
 $\therefore$ [B] ✓ (1 mark)

- 4 a gradient = $\frac{\text{rise}}{\text{run}}$
 $= \frac{2}{5}$ ✓ (1 mark)

- b y -intercept = 2 ✓ (1 mark)

- c The line $y = \frac{2}{5}x + 4$ is parallel to the given line with y -intercept 4.



- 5
-
- a $A(-2, 1), B(5, 2)$
 $m = \frac{\text{rise}}{\text{run}}$
 $= \frac{1}{7}$ ✓
 $D(2, -3), C(9, -2)$
 $m = \frac{\text{rise}}{\text{run}}$
 $= \frac{1}{7}$
 $m_{AB} = m_{DC}$
 $\therefore AB$ is parallel to DC ✓
(2 marks)

WORKED SOLUTIONS



CHECK YOUR SOLUTIONS

b $A(-2, 1), D(2, -3)$

$$m = \frac{\text{rise}}{\text{run}} = \frac{-4}{4} = -1 \quad \checkmark$$

$B(5, 2), C(9, -2)$

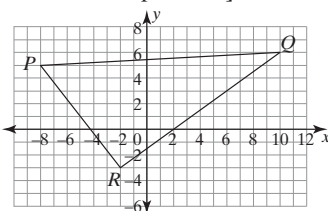
$$m = \frac{\text{rise}}{\text{run}} = \frac{-4}{4} = -1$$

$$m_{AD} = m_{BC}$$

$\therefore AD$ is parallel to BC $\checkmark$
(2 marks)

c $ABCD$ is a parallelogram. $\checkmark$
[Both pairs of opposite sides are parallel.] (1 mark)

6



[From the diagram if the triangle has a right angle it will be at R .]

$$m_{PR} = \frac{\text{rise}}{\text{run}} = \frac{-8}{6} = -\frac{4}{3} \quad \checkmark$$

$$m_{QR} = \frac{\text{rise}}{\text{run}} = \frac{9}{12} = \frac{3}{4} \quad \checkmark$$

$\therefore$ the triangle is right-angled $\checkmark$
 PR and QR are perpendicular because the gradients are negative reciprocals. $\checkmark$
(4 marks)

7

a $y = -4x + 3$
The gradient of this line is -4 . Line k is parallel to line l so it also has gradient -4 . $\checkmark$
Line k has y -intercept -1 .
So $m = -4$ and $b = -1$
The equation of line k is $y = -4x - 1$. $\checkmark$ (2 marks)

b Line m is perpendicular to line l so its gradient is $\frac{1}{4}$. $\checkmark$
Lines l and m meet on the y -axis. The y -intercept of line l is 3 so the y -intercept of line m is also 3. $\checkmark$
So $m = \frac{1}{4}$ and $b = 3$
The equation of line m is $y = \frac{1}{4}x + 3$ $\checkmark$ (3 marks)

8

a $A(7, -8), B(1, 1)$
 $m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-8)}{1 - 7} = \frac{9}{-6} = -\frac{3}{2}$ $\checkmark$ (1 mark)

b $B(1, 1), C(-3, 7)$
 $m_{BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 1}{-3 - 1} = \frac{6}{-4} = -\frac{3}{2}$ $\checkmark$ (1 mark)

c A, B and C are collinear. $\checkmark$
[They lie on the same straight line.] $\checkmark$
 AB and BC have the same gradient and B is a common point. $\checkmark$ (2 marks)
(Total: 25 marks)

PARALLEL AND PERPENDICULAR LINES

ADVANCED TEST

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1 a $y = 3x - 2$ has gradient 3
 $x + 3y - 4 = 0$
 $3y = -x + 4$
 $y = -\frac{1}{3}x + \frac{4}{3}$
This line has gradient $-\frac{1}{3}$. $\checkmark$
Now $3 \times -\frac{1}{3} = -1$
So the lines are perpendicular. $\checkmark$ (2 marks)

b $2x + y - 3 = 0$
 $y = -2x + 3$
gradient $= -2$ $\checkmark$
 $x + 2y + 5 = 0$
 $2y = -x - 5$
 $y = -\frac{1}{2}x - 2.5$
gradient $= -\frac{1}{2}$
So the lines are neither parallel nor perpendicular. $\checkmark$ (2 marks)

c $y = \frac{3}{4}x - 5$
gradient $= \frac{3}{4}$
 $3x - 4y - 2 = 0$
 $4y = 3x - 2$
 $y = \frac{3}{4}x - 0.5$
gradient $= \frac{3}{4}$ $\checkmark$
The gradients are equal so the lines are parallel. $\checkmark$ (2 marks)

2

a $l: y = 7 - x$
 $= -x + 7$
Gradient $= -1$ $\checkmark$
Parallel line has equation $y = -x + b$
It passes through $(2, 1)$.
So $1 = -2 + b$
 $b = 3$
The equation is $y = -x + 3$. $\checkmark$ (2 marks)

b The two lines intersect on the y -axis so have the same y -intercept.
Line l has y -intercept 7
Perpendicular line has gradient 1. $\checkmark$
Equation is $y = x + 7$ $\checkmark$ (2 marks)

3

$P(1, 8), Q(-4, 5)$
a $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 8}{-4 - 1} = \frac{-3}{-5} = \frac{3}{5}$ $\checkmark$ (1 mark)

WORKED SOLUTIONS



CHECK YOUR SOLUTIONS

b $PQ \perp QR$

$\therefore$ gradient of $QR = -\frac{5}{3}$
At R , $y = 0$

So R is the point $(x, 0)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-\frac{5}{3} = \frac{5 - 0}{-4 - x} \quad \checkmark$$

$$-\frac{5}{3} = -\frac{5}{4 + x}$$

$$4 + x = 3$$

$$x = -1$$

So R is the point $(-1, 0)$ $\checkmark$
(2 marks)

4 a $S(-10, -5)$, $T(-1, 7)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{ST} = \frac{7 - (-5)}{-1 - (-10)}$$

$$= \frac{4}{3} \quad \checkmark \quad (1 \text{ mark})$$

b $T(-1, 7)$, $U(15, -5)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{TU} = \frac{-5 - 7}{15 - (-1)}$$

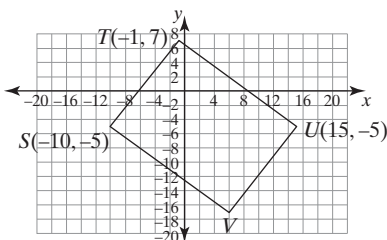
$$= -\frac{3}{4} \quad \checkmark$$

$$\text{Now } \frac{4}{3} \times -\frac{3}{4} = -1$$

So $ST \perp TU$

$\therefore \triangle STU$ is right-angled
at T . $\checkmark$ (2 marks)

c $V(6, -17)$ $\checkmark$ (1 mark)



d $STUV$ is a rectangle.

(A parallelogram with one angle a right-angle)

Distance $SU = 25$ units

T is 12 units above line SU . $\checkmark$

$$A = 2 \times \frac{1}{2} \times 25 \times 12$$

$$= 300$$

The area is 300 units². $\checkmark$
(2 marks)

5 $A(-2, 4)$, $B(6, 2)$

a midpoint = $\left(\frac{-2+6}{2}, \frac{4+2}{2}\right)$
 $= (2, 3)$ $\checkmark$ (1 mark)

b $m = \frac{y_2 - y_1}{x_2 - x_1}$
 $= \frac{2 - 4}{6 - (-2)}$
 $= -\frac{1}{4}$ $\checkmark$ (1 mark)

c Perpendicular line has gradient 4.

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 4(x - 2)$$

$$y - 3 = 4x - 8$$

$$y = 4x - 5 \quad \checkmark \quad (1 \text{ mark})$$

d $A(-2, 4)$, $C(-4, 2)$
midpoint = $(-3, 3)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2 - 4}{-4 - (-2)}$$

$$= 1$$

Perpendicular line

has $m = -1$ $\checkmark$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -1(x - (-3))$$

$$y - 3 = -x - 3$$

$$y = -x \quad \checkmark \quad (2 \text{ marks})$$

e At point of intersection:

$$4x - 5 = -x$$

$$5x = 5$$

$$x = 1 \quad \checkmark$$

When $x = 1$, $y = -1$

$\therefore$ the point of intersection
is $(1, -1)$. $\checkmark$ (2 marks)

f $B(6, 2)$, $C(-4, 2)$

midpoint = $(1, 2)$

Now both B and C lie on
the horizontal line $y = 2$

So the perpendicular line
will be vertical.

The equation of the
perpendicular bisector
will be $x = 1$. $\checkmark$ (1 mark)

g All three bisectors will
meet at the same point;

$(1, -1)$. $\checkmark$ (1 mark)

6 a $ax + by + c = 0$

$$by = -ax - c$$

$$y = -\frac{a}{b}x - \frac{c}{b} \quad \checkmark$$

(1 mark)

b Parallel lines have the same
gradient.

$$\text{So } m = -\frac{a}{b} \quad \checkmark \quad (1 \text{ mark})$$

c $5x - 3y + 7 = 0$

$$m = -\frac{5}{-3}$$

$$= \frac{5}{3}$$

So the gradient of any
parallel line is $\frac{5}{3}$. $\checkmark$ (1 mark)

d $m = \frac{5}{3}$, y -intercept = 0

$$\text{Equation is } y = \frac{5}{3}x$$

$$3y = 5x$$

$$5x - 3y = 0 \quad \checkmark \quad (1 \text{ mark})$$

e $m = \frac{b}{a}$

[The negative reciprocal.]
 $\checkmark$ (1 mark)

f $m = \frac{b}{a}$
 $= \frac{2}{4}$
 $= \frac{1}{2} \quad \checkmark \quad (1 \text{ mark})$

g $y = \frac{1}{2}x$

$$2y = x$$

$$x - 2y = 0 \quad \checkmark \quad (1 \text{ mark})$$

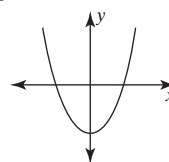
(Total: 35 marks)

FURTHER GRAPHS

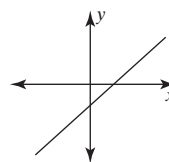
SKILLS CHECK

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1 a parabola



b straight line

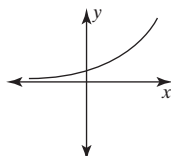


WORKED SOLUTIONS

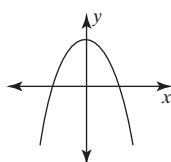


CHECK YOUR SOLUTIONS

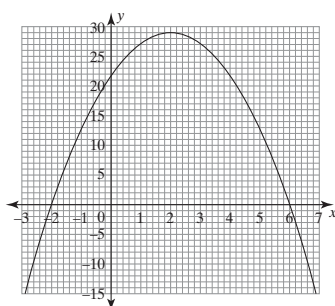
c exponential curve



d parabola



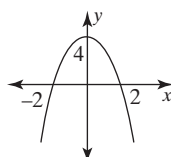
2



- a y-intercept = 22
- b When $x = 4$, $y = 22$
- c The parabola cuts the x -axis at $x = -2$ and $x = 6$. So the points are $(-2, 0)$ and $(6, 0)$.
- d The vertex is at $(2, 29)$.
- e $y = 13$ when $x = -1$ and $x = 5$.

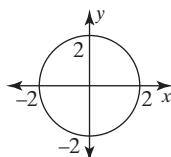
3

a



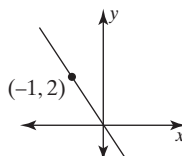
$y = 4 - x^2$
[parabola, 'upside down',
y-intercept = 4]

b



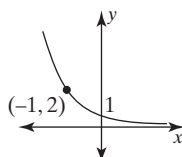
$x^2 + y^2 = 4$
[circle, centre $(0, 0)$,
radius 2]

c



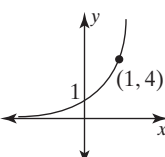
$y = -2x$
[gradient = -2,
y-intercept = 0]

d



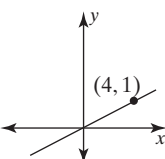
$y = 2^{-x}$
[exponential curve,
when $x = -1$, $y = 2$]

e



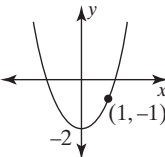
$y = 4^x$
[exponential curve,
when $x = 1$, $y = 4$]

f



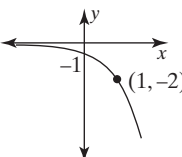
$y = \frac{x}{4}$
[gradient = $\frac{1}{4}$,
y-intercept = 0]

g



$y = x^2 - 2$
[parabola, y-intercept -2]

h



$y = -2^x$
[exponential curve, 'upside down']

4

a $y = 3 - x$ straight line
[$y = -x + 3$, gradient -1,
y-intercept 3]

b $y = 3x^2$ parabola

c $y = 3^x$ exponential curve

d $y = x^3$ cubic curve

e $y = x^2 + 3$ parabola

f $x^2 + y^2 = 3$ circle
[centre $(0, 0)$, radius $\sqrt{3}$]

g $y = \frac{x}{3}$ straight line

[$y = \frac{1}{3}x$, gradient $\frac{1}{3}$,
y-intercept 0]

h $y = -x^2 + 2$ parabola

i $y = 9 - x^2$ parabola

j $x + y = 9$ straight line
[$y = -x + 9$]

k $x^2 + y^2 = 9$ circle
[centre $(0, 0)$, radius 3]

l $y = 2^{-x}$ exponential curve

FURTHER GRAPHS

INTERMEDIATE TEST PAGE 51

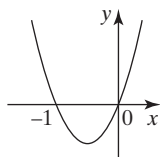
- 1 Equations of straight lines can be put in the form $y = mx + b$.
 $y = \frac{3}{x}$ is not of this form so it is not the equation of a straight line.
[The other equations can be rearranged:
A: $y = -x + 3$, B: $y = x$,
C: $y = \frac{1}{2}x + \frac{3}{2}$]
 $\therefore$ [D] ✓ (1 mark)
- 2 $x^2 + y^2 = 50$
Try each option.
 $(-1, -7)$: $(-1)^2 + (-7)^2 = 1 + 49 = 50$
 $(7, -1)$: $7^2 + (-1)^2 = 49 + 1 = 50$
 $(2, 6)$: $2^2 + 6^2 = 4 + 36 = 40$ (not 50)
 $(5, 5)$: $5^2 + 5^2 = 25 + 25 = 50$
The point that does not lie on the circle is $(2, 6)$.
 $\therefore$ [C] ✓ (1 mark)

WORKED SOLUTIONS



CHECK YOUR SOLUTIONS

3 $y = x^2 + x$



When $x = 0$, $y = 0^2 + 0 = 0$

When $x = -1$, $y = (-1)^2 + (-1) = 0$

The only possible graph is B. $\therefore$ [B] ✓ (1 mark)

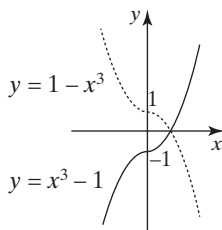
4 The equation of a parabola is a quadratic equation.

$$y = (x + 1)(x - 1) = x^2 - 1$$

This is the equation of a parabola.

$\therefore$ [A] ✓ (1 mark)

5 a From the graph there is one solution to the equation $x^3 - 1 = 1 - x^3$. ✓



[The two graphs have only one point of intersection.] (1 mark)

b $x^3 - 1 = 1 - x^3$

$$2x^3 = 2$$

$$x^3 = 1$$

$$x = 1 \quad [\sqrt[3]{1}] \quad \checkmark$$

(1 mark)

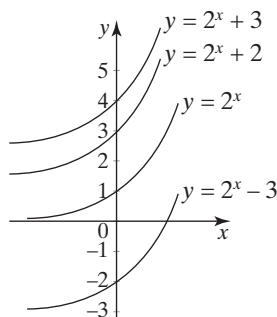
6 $x^2 + y^2 = 36$

$$x^2 + y^2 = 6^2$$

This circle, centre O , has radius 6. ✓

The distance from O to P is 8 units. So, the point P will lie outside the circle because the distance from O to P is greater than the radius of the circle. ✓ (2 marks)

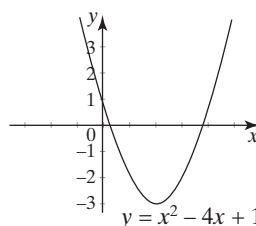
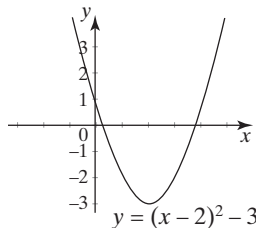
7 a The constant c moves the graph up or down the y -axis. ✓ (1 mark)



b $y = 2^x$ has y -intercept 1. $y = 2^x + 7$ will have y -intercept 8. $[1 + 7]$ ✓ (1 mark)

c $y = 2^x - 1$ will pass through the origin. [The graph $y = 2^x$ needs to move down 1.] $y = 2^x + c$ will pass through the origin when $c = -1$. ✓ (1 mark)

8 a The two graphs are exactly the same. ✓ (1 mark)



$$\begin{aligned} (x - 2)^2 - 3 &= (x - 2)(x - 2) - 3 \\ &= x^2 - 2x - 2x + 4 - 3 \\ &= x^2 - 4x + 1 \quad \checkmark \end{aligned}$$

The two expressions are in different forms but when simplified are the same so the graphs are also the same. ✓ (3 marks)

(Total: 15 marks)

FURTHER GRAPHS ADVANCED TEST

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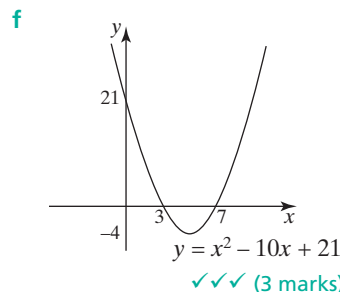
1 a $y = x^2 - 10x + 21$ is a parabola. ✓ (1 mark)

b When $x = 0$, $y = 21$. The y -intercept is 21. ✓ (1 mark)

c When $y = 0$, $x^2 - 10x + 21 = 0$
 $(x - 3)(x - 7) = 0$
 $x = 3$ or $x = 7$
The x -intercepts are 3 and 7. ✓✓ (2 marks)

d The equation of the axis of symmetry is $x = 5$. [It passes through the midpoint of the x -intercepts.] ✓ (1 mark)

e When $x = 5$, $y = 5^2 - 10 \times 5 + 21 = -4$. The minimum value is -4 . ✓ (1 mark)



2 a Both curves are the same size and shape as $y = x^2$. $y = x^2 + 2$ has moved up two units from the position of $y = x^2$ and $y = x^2 - 3$ has moved down 3 units. ✓ (1 mark)

b Both curves are the same size and shape as $y = x^2$. $y = (x + 2)^2$ has moved two units across to the left from the position of $y = x^2$ and $y = (x - 3)^2$ has moved three units to the right. ✓ (1 mark)

c Both curves are the same size and shape but $y = -x^2$ is upside down (concave down). ✓ (1 mark)

WORKED SOLUTIONS



CHECK YOUR SOLUTIONS

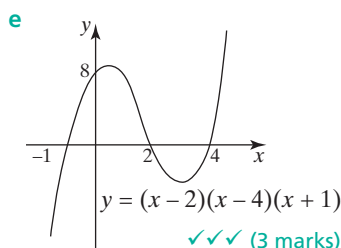
- d $y = 2x^2$ and $y = \frac{1}{2}x^2$ have the same axis of symmetry and minimum point as $y = x^2$ but $y = 2x^2$ is thinner than $y = x^2$ and $y = \frac{1}{2}x^2$ is wider. ✓ (1 mark)

- 3 a $y = (x + 2)^2 - 3$ has a minimum turning point at $(-2, -3)$ ✓✓ (2 marks)
b $y = 5 - (3 - x)^2$ has a maximum turning point at $(3, 5)$ ✓✓ (2 marks)

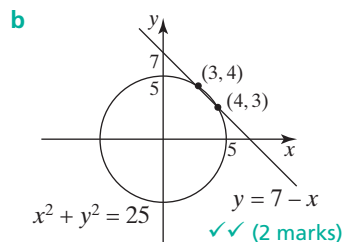
- 4 The curve $(x - 1)^2 + (y + 3)^2 = 16$ is a circle of radius 4 units. ✓
The centre is at $(1, -3)$. ✓ (2 marks)

- 5 a If n is even, $y = x^n$ will be symmetrical about the y -axis. The y -axis will be an axis of symmetry. ✓ (1 mark)
b If n is odd, $y = x^n$ will have point symmetry about the origin. If rotated through 180° about $(0, 0)$ it will be exactly the same. ✓ (1 mark)

- 6 $y = (x - 2)(x - 4)(x + 1)$.
a When $x = 0$,
 $y = (0 - 2)(0 - 4)(0 + 1)$
 $= 8$ ✓ (1 mark)
b When $y = 0$,
 $(x - 2)(x - 4)(x + 1) = 0$
 $x = 2$ or $x = 4$ or $x = -1$ ✓ (1 mark)
c If x has a large positive value, y will also have a large positive value. ✓ (1 mark)
d If x has a large negative value, y will also have a large negative value. ✓ (1 mark)



- 7 a $x^2 + y^2 = 25$ [i]
 $y = 7 - x$ [ii]
Substitute [ii] into [i]
 $x^2 + (7 - x)^2 = 25$
 $x^2 + 49 - 14x + x^2 = 25$
 $2x^2 - 14x + 24 = 0$
 $x^2 - 7x + 12 = 0$
 $(x - 3)(x - 4) = 0$
 $x = 3$ or $x = 4$ ✓
Substitute into [ii]
When $x = 3$, $y = 7 - 3$
 $= 4$
When $x = 4$, $y = 7 - 4$
 $= 3$ ✓ (2 marks)



- 8
-
- a $y = 2^x - 3$
When $x = 0$,
 $y = 2^0 - 3$
 $= -2$
So $a = -2$ ✓ (1 mark)
b $y = 2^x$ cuts the y -axis at $y = 1$
 $y = 2^x - 3$
When $y = 1$,
 $1 = 2^x - 3$
 $4 = 2^x$
But $2^2 = 4$
So $x = 2$
 $\therefore b = 2$ ✓ (1 mark)
c $y = 2^x - 3$ cuts the x -axis at $x = c$
When $x = c$ and $y = 0$
 $2^c - 3 = 0$
 $2^c = 3$
Now $2^1 = 2$ and $2^2 = 4$
So c lies between 1 and 2.

$2^{1.5} = 2.828\dots$
 $2^{1.6} = 3.031\dots$
So c lies between 1.5 and 1.6
 $2^{1.58} = 2.989\dots$
 $2^{1.59} = 3.010\dots$
So c lies between 1.58 and 1.59
 $2^{1.584} = 2.997\dots$
 $2^{1.585} = 3.000\dots$
So c lies between 1.584 and 1.585
 $2^{1.5845} = 2.999\dots$
So $c = 1.585$ to three decimal places. ✓ (1 mark)
(Total: 35 marks)